ON THE THEORY OF SUM-EQUATIONS

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1. **Introduction.** A system of linear equations is called a *sum-equation system* if it has the form

$$(1.1) \qquad a_{n0}x_n + a_{n1}x_{n+1} + \cdots = c_n \qquad (n = 0, 1, \cdots),$$

with $a_{n0} \neq 0$. All quantities are assumed to be complex numbers. Let $\{A_n(t)\}$ be defined by

$$(1.2) \qquad A_n(t) = \sum_{s=0}^{\infty} a_{ns}t^s \qquad (n = 0, 1, \cdots),$$

and define the *type* of a sequence $\{y_n\}$ as the number $((y_n))$ given by

$$(1.3) \qquad ((y_n)) \equiv \limsup_{n\to\infty} |y_n|^{1/n}.$$

Suppose the functions $\{A_n(t)\}$ are analytic in $|t| < q$, and that $((c_n)) = \delta < q$, so that the function

$$(1.4) \qquad C(t) = \sum_{0}^{\infty} c_n t^n$$

has the radius of convergence $1/\delta$.

Let functions $\{H_{k,j}(t)\}$ $(j=0, 1, \cdots, k-1;\ k=0, 1, \cdots)$ be defined by

$$(1.5) \qquad H_{k,j}(t) = c_j + c_{j+k}t^k + c_{j+2k}t^{2k} + \cdots,$$

and let

$$(1.6) \qquad \Delta_k(t; H) = \begin{vmatrix} H_{k,0}(1/t) & (1/t)H_{k,1}(1/t) & \cdots & (1/t^{k-1})H_{k,k-1}(1/t) \\ A_0(\omega_k t) & \omega_k A_1(\omega_k t) & \cdots & \omega_k^{k-1}A_{k-1}(\omega_k t) \\ \cdot & \cdot & \cdots & \cdot \\ A_0(\omega_k^{k-1}t) & \omega_k^{k-1}A_1(\omega_k^{k-1}t) & \cdots & \omega_k^{(k-1)(k-1)}A_{k-1}(\omega_k^{k-1}t) \end{vmatrix},$$

where

$$(1.7) \qquad \omega_k = e^{2\pi i/k} \qquad [i = (-1)^{1/2}].$$

The functions $H_{k,j}(t)$ are analytic in $|t| < 1/\delta$, the function $\Delta_k(t)$ given by

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