INNER DERIVATIONS OF NON-ASSOCIATIVE ALGEBRAS

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In this note we propose a definition of inner derivation for non-associative algebras. This definition coincides with the usual one for Lie algebras, and for associative algebras with no absolute right (left) divisor of zero. It is well known that all derivations of semi-simple associative or Lie algebras over a field of characteristic zero are inner.

Recent correspondence with N. Jacobson has revealed that a number of the ideas in this note duplicate some of his current researches.[1] In particular, he has shown that every derivation of a semi-simple non-associative algebra (that is, direct sum of simple algebras) with a unity quantity over a field of characteristic zero is inner in this sense.

1. Preliminaries. A *derivation* of a non-associative algebra $\mathfrak{A}$ over a field $\mathfrak{F}$ is a linear transformation D on $\mathfrak{A}$ satisfying

$$(xy)D = x(yD) + (xD)y \tag{1}$$

for all x, y in $\mathfrak{A}$. It is known [2][2] that the set $\mathfrak{D}$ of all derivations of $\mathfrak{A}$ is a Lie algebra over $\mathfrak{F}$ if multiplication in $\mathfrak{D}$ is defined by

$$[D_1, D_2] = D_1D_2 - D_2D_1 \tag{2}$$

where D_1D_2 is the ordinary (associative) multiplication of linear transformations. $\mathfrak{D}$ is called the *derivation algebra* of $\mathfrak{A}$.

If we write R_y for the *right multiplication*

$$x \to xy = xR_y \qquad \text{for all } x \text{ in } \mathfrak{A}$$

and L_x for the *left multiplication*

$$y \to xy = yL_x \qquad \text{for all } y \text{ in } \mathfrak{A},$$

the definition (1) is seen to be equivalent to either one of

$$[R_y, D] = R_{yD} \qquad \text{for all } y \text{ in } \mathfrak{A} \tag{3}$$

or

$$[L_x, D] = L_{xD} \qquad \text{for all } x \text{ in } \mathfrak{A}. \tag{4}$$

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[1] N. Jacobson, *Derivation algebras and multiplication algebras of semi-simple Jordan algebras*, to appear in Ann. of Math.

[2] Numbers in brackets refer to the references cited at the end of the paper.