A THEOREM ON QUADRATIC FORMS OVER THE RING OF 2-ADIC INTEGERS[1]

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1. Introduction. The theory of the equivalence of quadratic forms over the ring of 2-adic integers is considerably more difficult than the corresponding theory for forms over the p-adic integers, p an odd prime, and has only recently been worked out to a comparable degree. A problem still not completely solved is Witt's cancellation theorem: namely, if f, g and h are forms such that g and h have no variables in common with f, then $f+g$ equivalent to $f+h$ implies g equivalent to h. This theorem, though true for p-adic integers when p is odd, does not always hold when p is even. We give here a case when it does hold (see theorem below). While this theorem follows almost immediately from results in a paper by Jones [1, Theorems 2 and 6][2] it seems worthwhile in view of the rather long arguments of the theorems there to give an independent proof, especially since it in turn can be used to shorten some of the proofs of that paper.

2. Proof of the theorem. We denote by capital italic letters matrices over $R(2)$, the ring of 2-adic integers, while small italic letters with the exception of f, g and h will stand for numbers in $R(2)$. We shall consider only forms whose symmetric matrices have elements in $R(2)$. A matrix is *unimodular* if its elements are in $R(2)$ and its determinant a unit of $R(2)$. A form is called unimodular if its symmetric matrix is unimodular.

THEOREM. *Let f_1 and f_2 be two equivalent unimodular forms over $R(2)$ in $x_1, x_2, \cdots, x_n$, g a nonsingular form over $R(2)$ in $x_{n+1}, x_{n+2}, \cdots, x_{n+s}$ and h a nonsingular form over $R(2)$ in $x_{n+1}, x_{n+2}, \cdots, x_{n+t}$. If there is a matrix over $R(2)$ taking f_1+2g into f_2+2h, then there is one over $R(2)$ taking g into h; if $s=t$ and the former matrix is unimodular, then so is the latter.*

PROOF. Since f_1 and f_2 are equivalent we may take f_1+2g into f_2+2g by a unimodular transformation and thus we set $f_1=f_2=f$. By [1, Lemma 1] f is equivalent to either a diagonal form or a sum of binary forms. If the latter is the case, then x^2+f will be equivalent to

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[2] Numbers in brackets refer to the bibliography at the end of the paper.