ON CONTINUOUS CURVES IRREDUCIBLE ABOUT COMPACT SETS

GAIL S. YOUNG, JR.

Some years ago, Leo Zippin [4][1] raised the following question: Given a compact set K in a locally connected complete metric space, S, it is known that S contains a compact continuum M irreducible about K. What conditions on K are necessary and sufficient that M may be chosen locally connected? He showed that a necessary condition is that K be a curve set; that is, the nondegenerate components of K are locally connected and form a null sequence. For K 1-dimensional, he proved the condition is also sufficient, and it was conjectured that it is always sufficient. Later Martin Ettlinger [3] gave other results tending to support this. In this note I give a partial solution.

THEOREM. *If K is a compact curve set in a convex metric space S, then S contains a locally connected compact continuum irreducible about K.*

The recent announcement by R. H. Bing [1] of a proof that every finite-dimensional Peano space has a convex metric makes this result appear less special than it might seem otherwise.

PROOF. Zippin has shown in [4] that the proposition need be proved only for the case where K is a special curve set, which is a curve set with only a countable number of components, all but one being points forming a sequence converging to that one. His argument is stated for S compact, but, however, is valid under our hypothesis. Hence, we may suppose K is the sum of a sequence $\{x_n\}$ of distinct points and a locally connected continuum C. The sequence x_n can be chosen so that $d(x_n, C) \geqq d(x_{n+1}, C)$. For each n, let y_n denote a point of C such that $d(x_n, y_n) = d(x_n, C)$. Let T_1 denote a straight-line interval from x_1 to y_1. Let x_{n_1} be the first point of x_n not in T_1, and let T_2 denote a straight-line interval joining x_{n_1} and y_{n_1}. If $T_1 \cdot T_2$ is not empty, replace the interval of T_2 irreducible about $y_{n_1} + T_1 \cdot T_2$ by the arc of T_1 irreducible about $y_1 + T_1 \cdot T_2$, and let T_2 now denote this new arc. It is easy to see from the triangle inequality and the convexity of T_1 and the original T_2 that this does not change the length of T_2. We now have that $T_1 \cdot T_2$ is either empty or is connected and contains y_1. Let x_{n_2} be the first point of x_n not in

Presented to the Society, April 17, 1948; received by the editors March 29, 1948.

[1] Numbers in brackets refer to the bibliography.