REMARKS ON CYCLIC ADDITIVITY

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1. Introduction. For the purposes of this discussion suppose that X and Y are topological spaces while G is a commutative, topological semi-group (with zero element) which, as a space, is Hausdorff. In other words, each pair (g_1, g_2) of elements in G uniquely determines an element (g_1+g_2) in G; the operation $+$ is associative and commutative; there is a unique element 0 such that $g \in G$ implies $g+0=g$; finally, G is a Hausdorf space and the operation $+$ provides a mapping (=continuous transformation) from the product space $G\times G$ into G. Obviously, topological groups, and the space of non-negative real numbers compactified by the addition of ∞, with the operation $+$ meaning addition, and the convention that $a+\infty=\infty+a=\infty$, provide examples of such semi-groups.

It will be said that lm is a *Peanian factorization* of a mapping $f: X \rightarrow Y$ if and only if there are mappings $m: X \rightarrow \mathfrak{X}$ and $l: \mathfrak{X} \rightarrow Y$ such that $\mathfrak{X}$ is a Peano space and the composition lm is f. The space $\mathfrak{X}$ is called the *middle space* of the Peanian factorization lm of f.

Let $\boldsymbol{F}$ be the class of mappings $f: X \rightarrow Y$ each of which has at least one Peanian factorization, and suppose that γ is a transformation from $\boldsymbol{F}$ into G.

For each Peano space $\mathfrak{X}$ let $\mathcal{E}(\mathfrak{X})$ be the class of true cyclic elements of $\mathfrak{X}$. (See Whyburn [6] for the Peano space theory involved in this paper.)[2] If $\mathfrak{C} \in \mathcal{E}(\mathfrak{X})$ there is a unique monotone retraction $r_{\mathfrak{C}}: \mathfrak{X} \rightrightarrows \mathfrak{C}$. (The double arrow indicates that $r_{\mathfrak{C}}(\mathfrak{X})=\mathfrak{C}$.) If $f \in \boldsymbol{F}$ and lm is a Peanian factorization of f with middle space $\mathfrak{X}$, while $\mathfrak{C} \in \mathcal{E}(\mathfrak{X})$, then define $f_{\mathfrak{C}}=l r_{\mathfrak{C}} m$.

It is the object of this paper to investigate the statement

$$\gamma(f) = \sum \gamma(f_{\mathfrak{C}}), \qquad \mathfrak{C} \in \mathcal{E}(\mathfrak{X}) \tag{1}$$

where the equality means that for each neighborhood U of $\gamma(f)$ there is a finite subclass $\mathcal{J}(U)$ of $\mathcal{E}(\mathfrak{X})$ such that if $\mathcal{J}$ is any finite subclass of $\mathcal{E}(\mathfrak{X})$ containing $\mathcal{J}(U)$ then $U \ni \sum \gamma(f_{\mathfrak{C}})$, $\mathfrak{C} \in \mathcal{J}$, it being understood that addition over an empty class yields 0.

In the event that (1) holds for each $f \in \boldsymbol{F}$ and for each Peanian factorization of f, then γ is said to be *cyclicly additive*.

Cyclic additivity theorems of a weaker type have been considered

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[2] Numbers in brackets refer to the bibliography at the end of the paper.