A CLASS OF TOPOLOGICAL SPACES

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1. Introduction. It is a classical theorem of set-theoretical topology that a one-to-one continuous mapping of a bicompact Hausdorff space onto a Hausdorff space is a homeomorphism.[1] Stated in somewhat different terms, this theorem asserts that any bicompact Hausdorff topology on a given set E is a *minimal* Hausdorff topology. If $\mathcal{B}$ is the family of open sets in this topology, then no proper subfamily of $\mathcal{B}$ can be the family of open sets for a Hausdorff topology on E. When this phenomenon is observed, a number of questions immediately present themselves:

(1) Under what conditions will a minimal Hausdorff space be bicompact?

(2) Are there any minimal Hausdorff spaces which are not bicompact?

(3) Is there any simple way of characterizing those topological spaces which have one-to-one continuous images which are bicompact Hausdorff spaces?

Question (1) was answered completely by Katětov [6, p. 40], who proved that a Hausdorff space is bicompact if and only if it is minimal and satisfies the Urysohn separation axiom (that is, every pair of distinct points possess neighborhoods whose closures are disjoint).

Question (2) can be answered in the affirmative by modifying a space constructed by Urysohn (see [2, p. 22]). Let the space X_0 be defined as the set of all points (x, y) in the Euclidean plane such that $0<x^2+y^2\leqq 1$, together with two adjoined points p^+ and p^-. Let neighborhoods of all points (x, y) in X_0 be the usual Euclidean neighborhoods; let $U_n(p^+)$ be $p^+\cup E[(x, y),\ 0<x^2+y^2<1/n,\ y>0]$; and let $U_n(p^-)$ be $p^-\cup E[(x, y),\ 0<x^2+y^2<1/n,\ y<0]$. As the index n assumes all positive integral values, the neighborhoods $U_n(p^+)$ and $U_n(p^-)$ describe a complete family of neighborhoods for p^+ and p^-. It is obvious that, under this definition, X_0 forms a Hausdorff space which fails to satisfy the Urysohn separation axiom. It is easy to prove, moreover, that X_0 is a minimal Hausdorff space. Katětov [6,

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[1] See, for example [1, p. 95, Satz III]. It is interesting to observe that this result was stated in 1893 by Jordan for bounded closed subsets of n-dimensional Euclidean space (see [5, p. 53]). Numbers in brackets refer to the references cited at the end of the paper.