ON A THEOREM OF REPRESENTATION

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Introduction. The main result of this paper is the characterization of the linear ring (BC) of all bounded continuous real functions defined in a locally compact Hausdorff space which vanish at infinity (see Theorem 5). This is done by making use of some of the properties of the functional $\phi(x)$ = greatest lower bound of x, defined for the elements of (BC) (see §. 1 and §. 8).

In this study we noticed that ϕ could give to the linear ring (BC) both its structures of natural partial order and topology. We had then the idea of seeing when this could be done in general, that is to say, when is it true that for a partially ordered normed linear ring or a normed linear lattice there is a functional which could give both the structures of partial order and topology of the space. The rest of our paper deals with such a question.

For the demonstration of our theorem 5 we made very much use of Professor Stone's paper, *A general theory of spectra*, I, the reading of which we recommend for having a clear understanding of this one.

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1. **Definition of S-space.** A linear space with real scalars V is said to be *an S-space with respect to a real function $\phi(x)$*, defined in V, if and only if

(S1) $\phi(x) \leqq 0$ for any $x \in V$;

(S2) $\phi(x+y) \geqq \phi(x)+\phi(y)$ for any $x \in V$ and any $y \in V$;

(S3) $\phi(\lambda x) = \lambda\phi(x)$ for any $x \in V$ and any real number $\lambda \geqq 0$;

(S4) $\phi(0) = 0$;

(S5) $\phi(x)+\phi(-x) = 0$ implies $x = 0$ $(x \in V)$;

(S6) the equations in the unknown ω

$$\text{(E)} \qquad \phi(\omega) = 0, \qquad \phi(\omega - x) = 0 \qquad (x \in V)$$

have a solution $x^+ \in V$ which is such that any other solution $x' \in V$ of (E) satisfies the condition $\phi(x'-x^+) = 0$;

(S7) $\lim_{n,m\to\infty} \phi(x_n - x_m) = 0$ implies the existence of an element $x \in V$ such that $\lim_{n\to\infty} \phi(x_n - x) = \lim_{n\to\infty} \phi(x - x_n) = 0$ ($\{x_n\}$ is a sequence of elements of V).

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