THE SOLUTION OF LINEAR INTEGRAL EQUATIONS BY MEANS OF WIENER INTEGRALS

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1. Introduction. In this paper, we obtain expressions for the solution, resolvent kernel, and Fredholm determinant of the integral equation

$$z(t) = x(t) + \int_0^1 K(t, s)x(s)ds$$

in terms of Wiener integrals. Besides appealing to the general interest which is always inherent in the relating of two apparently diverse fields, these results are of possible importance in two ways: (a) Though the Wiener integrals involved can be evaluated at present only when the kernel takes on certain relatively simple forms, it may be possible in the future to obtain at least approximate evaluations which will in turn offer approximate solutions of the integral equation which may converge faster than the Fredholm solution. Here we might mention specifically the case where the integral equation contains a parameter of large absolute value; (b) As a means of evaluating Wiener integrals in terms of the known (Fredholm) solution of the integral equation.

The Wiener integral is based on a measure defined by Wiener [1][1] on the space C of all real functions $x(t)$ continuous on the interval $0 \leqq t \leqq 1$ and vanishing at $t=0$. Cameron and Martin have investigated its properties and have, in particular, discovered how it transforms under translations [2] and linear transformations [3]. They have also been able to express the solution of a class of nonlinear integral equations in terms of limits of Wiener integrals [4]. We shall obtain our results by using their theorems on translations and linear transformations.

2. The basic solution. Given the integral equation

$$z(t) = F[x \mid t], \tag{2.1}$$

where

$$F[x \mid t] = x(t) + \int_0^1 K(t, s)x(s)ds, \tag{2.2}$$

Presented to the Society, April 26, 1947; received by the editors February 5, 1948, and, in revised form, April 5, 1948.

[1] Numbers in brackets refer to the references cited at the end of the paper.