

BOOK REVIEWS

Finite differences and difference equations in the real domain. By Tomlinson Fort. London, Oxford University Press, 1948. 7+251 pp. \$8.00.

The subject-matter of this book, as the title indicates, falls into two general categories; roughly half of the book is devoted to each of them.

In the first category, *finite differences*, are the following topics: difference operators and their elementary properties; the problem (in its most elementary form) of the *sum* of a function; the Bernoulli polynomials and numbers, and the Euler-Maclaurin and Euler summation formulas, and generalizations of all these concepts (the method of generalization is a brief and very inclusive one due to the author); numerical differentiation; interpolation formulas; numerical integration.

The second category, *difference equations in the real domain*, is made up almost exclusively of topics in the theory of the linear *recurrent relation*, that is, of the linear difference equation where the domain of the independent variable is a set of *integers*. The section of the book devoted to these topics consists in large part of published and unpublished research work of the author. For the homogeneous n th-order linear recurrent relation the concept of *fundamental system* of solutions is studied, and for the nonhomogeneous equation the method of variation of constants is described; special techniques for the case of constant coefficients are treated. Study is made of the question of the number of linearly independent solutions of that difference system which consists of a linear recurrent relation together with n linear boundary conditions, and the concept of a *Green's function* for an incompatible system is introduced.

For the *second order* linear recurrent relation, with coefficients dependent on a parameter, a Sturm-Liouville theory is developed. The manner in which the rapidity of oscillation of the solutions varies with the parameter is studied; the existence of characteristic values and characteristic functions is proved; the orthogonality of characteristic functions (orthogonality defined in terms of finite sums instead of integrals) is shown; and the expansion of an arbitrary function in terms of characteristic functions is obtained. Application is made to the problem of the vibration of a mass-less string loaded with n particles, and the vibration of a material string is studied as a limiting case. For second-order recurrent relations with periodic coefficients