RECENT PROGRESS IN THE GOLDBACH PROBLEM

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1. Introduction. The problem under consideration had its origin in a letter written by Goldbach to Euler in 1742 [4].[1] In the letter, Goldbach made two conjectures concerning the representation of integers as a sum of primes. They are equivalent to

(A) *every even integer greater than* 2 *is a sum of two primes,*
and
(B) *every integer greater than* 5 *is a sum of three primes.*

The two conjectures are, of course, equivalent. If $2n-2=p_1+p_2$, then $2n=p_1+p_2+2$ and $2n+1=p_1+p_2+3$. Conversely, if $2n=p_1+p_2+p_3$ one of the primes must be 2 and $2n-2=p_1+p_2$.

An impressive collection of numerical evidence indicating the truth of the conjectures has accumulated in the years since Goldbach's letter was written, but is it not known to this day whether the conjectures are true or false. What progress has been made towards the solution of the problem has been through two principal methods of attack.

The first of these is the sieve method (see §§2 and 3) due originally to Brun [1], and improved by Rademacher [16], Esterman [5], Ricci [17], [18], and Buchstab [2], [3]. The best result by this method, due to Buchstab in 1940, is

every sufficiently large even integer is a sum of two integers, each having at most four prime factors.

The sieve method has also been used in combination with results on the density of sequences of integers. (See §4.) Contributions have been made by Schnirelmann [20], Landau [12], [13], Heilbronn, Landau, Scherk [7], and Ricci [19]. Schnirelmann proved that

every integer >1 *is a sum of a finite number of primes.*

The best result, due to Ricci in 1937, a minor improvement on the Landau, Heilbronn, Scherk result of 1936, is

every sufficiently large integer is a sum of at most 67 *primes.*

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[1] Numbers in brackets refer to the references cited at the end of the paper.