and hence (6) holds for every sufficiently large fixed k. Now put in (5) $n=N_\nu=n_\nu-a_1a_2\cdots a_t$. Then for every fixed M

$$1 \geqq r_0 u_{N_\nu} + r_1 u_{N_\nu-1} + \cdots + r_M u_{N_\nu-M}.$$

As $\nu\to\infty$ all terms $u_{N_\nu-k}\to\lambda$ and hence

$$1 \geqq \lambda(r_0 + r_1 + \cdots + r_M)$$

or $\lambda \leqq 1/m$ (with $\lambda=0$ if $\sum r_n=\infty$).

If $m<\infty$ we can use a similar argument for $\mu=\liminf u_n$ to show that $\mu\geqq 1/m$. This proves the theorem.

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A CONSISTENCY THEOREM

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1. Introduction. Of primary importance in a theory of representation of functions by series which do not necessarily converge is its consistency theorem, which states that if a series which represents a function F converges to a function Φ, then $F\equiv\Phi$. Such a theorem for asymptotic representation in a strip region of a function by Dirichlet series with a certain logarithmic precision, an idea introduced by Mandelbrojt [1],[2] is the subject matter of this note. From it follow similar theorems for less general extensions of the idea of asymptotic series. The method consists in using the proof of the fundamental theorem in [1] to set up a homogeneous linear differential equation of infinite order with constant coefficients, which must be satisfied by the difference $F-\Phi$; then applying a method of Ritt to show that the only solution is identically zero.

The notation used by Mandelbrojt in [1] will be used here also. Let $\{\lambda_n\}$ be an increasing sequence of positive numbers ($0<\lambda_n\uparrow$). Denote by $N(\lambda)$, defined for $\lambda>0$, the *distribution function* of $\{\lambda_n\}$; that is, the number of terms in the sequence $\{\lambda_n\}$ less than λ; and by $D(\lambda)$ the *density function* of $\{\lambda_n\}$: $D(\lambda)=N(\lambda)/\lambda$. Let $D^\cdot$ represent the *upper density*: $D^\cdot=\limsup_{\lambda\to\infty}D(\lambda)$; and $D^\cdot(\lambda)$ the *upper density function* of $\{\lambda_n\}$: $D^\cdot(\lambda)=\text{l.u.b.}_{x\geqq\lambda}D(x)$; clearly $D^\cdot(\lambda)$ is continuous and decreases to $D^\cdot$ (unless $D^\cdot(\lambda)\equiv D^\cdot=\infty$).

Presented to the Society, April 17 1948; received by the editors March 15, 1948.

[1] The author is indebted to Professor Mandelbrojt for suggesting the problem considered in this note.

[2] Numbers in brackets refer to the bibliography at the end of the paper.