TOPOLOGICAL GROUPS AND GENERALIZED MANIFOLDS

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In a recent paper [4],[1] Montgomery showed that in a locally euclidean 3-dimensional group, any 2-dimensional closed subgroup is also locally euclidean. In this note we prove an analogous result for higher dimensions and more general spaces.

THEOREM. *Let G be a locally compact space which is both a topological group and an n-dimensional orientable generalized manifold. Let H be a closed connected $(n-1)$-dimensional subgroup. Then, if H carries a nonbounding $(n-1)$-cycle, H is also an orientable generalized manifold.*

The terminology used in the statement of this theorem, and in what follows, is that of our two previous papers on generalized manifolds [**1**, **2**], and we assume that the reader is familiar with them.

We make, however, one change. We find it convenient to define infinite cycles in the following way: We add to G an ideal point, g^+, taking as neighborhoods of g^+ those open subsets of G whose closures are not compact. Then $G^+=G\cup g^+$ is compact. Now an infinite cycle of G is defined to be a relative cycle of G^+ mod g^+. That this definition of infinite cycles is equivalent to the one used in [**2**] follows from Theorem 1.1 of [**2**].

LEMMA 1. *Given any neighborhood M of the unit element e of G, there is a neighborhood N of E such that for any infinite cycle Γ^k on H, $0\leqq k\leqq n-1$, and for any $g\in N$, $\Gamma^k\sim g\cdot\Gamma^k$ on $M\cdot H$.*

PROOF. Let $M_{n-1}\subset M$ have a compact closure. Choose a sequence

$$M_{n-1}\supset N_{n-1}\supset M_{n-2}\supset\cdots\supset M_0\supset N_0$$

such that N_i is obtained from M_i by the local connectedness of G in dimension i, and such that $M_i\cdot M_i\subset N_{i+1}$. Finally let N be such that $N\cdot N\subset N_0$.

Now let $g\in N$. To show that $\Gamma^k\sim g\cdot\Gamma^k$ on $M\cdot H$, it is sufficient to show that the coordinates of these cycles on the nerve of any covering U of G are homologous on $(M\cdot H)^+$. To this end, given a covering U, choose $U'<^*U$. Let X be the complement of the union of those sets of U' which contain g^+. Then X is a compact set. Let $X_1=\overline{M}_0\cdot X$ and $X_i=\overline{M}_{i-1}\cdot X_{i-1}$. Each X_i is a compact set.

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[1] Numbers in brackets refer to the bibliography at the end of the paper.