A GENERALIZATION OF A THEOREM ON LINEAR DIFFERENTIAL EQUATIONS

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1. **Introduction.** It is well known that the number of independent solutions of a linear homogeneous differential equation equals its order, and the general solution is a linear combination of these independent solutions. The object of this paper is to prove that in the general case, that is, the derivation is defined in an abstract field (not necessarily commutative), the number of independent solutions does not exceed the order of the equation.

Applying this theorem to the case of inner derivation one obtains a new proof of a theorem due to Artin and Whaples.[1] Another application concerning cyclic fields will be given elsewhere.

2. **Abstract linear differential equations.** F will denote a field (not necessarily commutative) with an automorphism S: $a \rightarrow aS$. A right S-derivation D of F is a mapping of F into a part of itself satisfying:[2]

$$(1)\quad (a+b)D = aD + bD, \qquad (2)\quad (ab)D = (aS)(bD) + (aD)b.$$

If condition (2') $(ab)D = a(bD) + (aD)(bS)$ holds instead of (2), D will be called a left S-derivation. The constants of the derivation (elements of F whose derivatives are zero) form a subfield C.

REMARK. Given a left S-derivation in F, one can define a right S-derivation in a field F^* which is anti-isomorphic to F. Hence any theorem about right derivation can be translated into a theorem on left derivation by suitable changes of "right" into "left."

As usual we denote $a^{(0)} = a$, $a' = aD$, $\cdots$, $a^{(n)} = (a^{(n-1)})D$. An equation:

$$(1)\qquad a_n z^{(n)} + a_{n-1} z^{(n-1)} + \cdots + a_0 z^{(0)} = 0, \qquad (a_n \neq 0,\ a_i \in F)$$

is called a right homogeneous linear differential equation (r.e.) of order n. When the $z^{(i)}$ are written on the left the equation is a left differential equation (l.e.).

THEOREM 1. *The elements of F which are solutions of the r.e.* (1) *form a right C-module of dimension at most n.*

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[1] Artin and Whaples, *The theory of simple rings*, Amer. J. Math. vol. 65 (1943) pp. 87–107.

[2] In the present paper operators are written multiplicatively on the right of the element on which they operate.