ON THE DIFFERENCE OF CONSECUTIVE PRIMES

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The present paper contains some elementary results on the difference of consecutive primes. Theorem 2 has been announced in a previous paper.[1] Also some unsolved problems are stated.

Let $p_1=2$, $p_2=3, \cdots, p_k, \cdots$ be the sequence of consecutive primes. Put $d_k=p_{k+1}-p_k$. We have:

THEOREM 1. *There exist positive real numbers* c_1 *and* c_2, $c_1<1, c_2<1$, *such that for every* n *the number of* k'*s satisfying both*

$$d_{k+1} > (1 + c_1)d_k, \qquad k \leqq n, \tag{1}$$

and the number of l'*s satisfying both*

$$d_{l+1} < (1 - c_1)d_l, \qquad l \leqq n, \tag{2}$$

are each greater than $c_2 n$.

We shall prove Theorem 1 later. From Theorem 1 we easily deduce:

THEOREM 2. *For every* t *and all sufficiently large* n *the number of solutions in* k *and* l *of each of the two sets of inequalities*

$$\left(\frac{p_{k+1}^t + p_{k-1}^t}{2}\right)^{1/t} > p_k, \ k \leqq n; \qquad \left(\frac{p_{l+1}^l + p_{l-1}^t}{2}\right)^{1/t} < p_l, \ l \leqq n, \tag{3}$$

is greater than $(c_2/2)n$.

Let ϵ be sufficiently small but fixed. It is well known that $p_n<2\cdot n \log n$. Thus the number of $k\leqq n$, with $p_{k+1}>(1+\epsilon)p_k$, is less than $c \log n$. Hence it follows from Theorem 1 that the number of k's satisfying

$$p_{k+1} < (1 + \epsilon)p_k, \qquad d_k > (1 + c_1)d_{k-1}, \qquad k \leqq n, \tag{4}$$

is greater than $(c_2/2)n$. A simple calculation now shows that the primes satisfying (4) also satisfy the first inequality of (3) if $\epsilon=\epsilon(c_1)$ is chosen small enough. The second inequality of (3) is proved in the same way, which proves Theorem 2.

Further, we obtain, as an immediate corollary of Theorem 1, that[2]

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[1] P. Erdös and P. Turán, *Some new questions on the distribution of primes*, Bull. Amer. Math. Soc. vol. 54 (1948) pp. 371–378.

[2] This result was also stated in the above paper.