A NOTE ON THE OPERATORS OF BLASCHKE AND PRIVALOFF

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Let $f(P)$ be a function of a point $P \equiv P(x, y)$ in Euclidean 2-space. Let $L(f; P; r)$, $A(f; P; r)$ be the mean values of $f(P)$ on the perimeter and on the interior, respectively, of a circle of center P and radius r, that is,

$$L(f; P; r) = \frac{1}{2\pi r} \int_{C(P;r)} f(Q) ds_Q,$$

$$A(f; P; r) = \frac{1}{\pi r^2} \iint_{D(P;r)} f(Q) dQ$$

where $C(P; r)$, $D(P; r)$ are the perimeter and interior, respectively, of the circle with center P and radius r. The operators

$$\nabla_p f(P) = \lim_{r \to 0} \frac{4}{r^2} [L(f; P; r) - f(P)],$$

$$\nabla_a f(P) = \lim_{r \to 0} \frac{8}{r^2} [A(f; P; r) - f(P)]$$

have been defined by Blaschke and Privaloff, respectively. The following are a few of the results which have been obtained by these and other investigators.

THEOREM A [1, 2].[1] *If $f(P)$ has continuous second partial derivatives, then $\nabla_p f(P)$, $\nabla_a f(P)$ exist, and*

$$\left(\frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2}\right)_P \equiv \nabla^2 f(P) = \nabla_p f(P) = \nabla_a f(P).$$

THEOREM B [1]. *If* (i) *$f(P)$ is continuous on a circle $\overline{D}(Q; r)$,* (ii) *$\nabla_p f(P)$ exists on the interior, $D(Q; r)$, then*

$$\frac{4}{r^2} [L(f; Q; r) - f(Q)]$$

lies between the upper and lower bounds of $\nabla_p f(P)$ on $D(Q; r)$.

THEOREM C [3, 4]. *If $u(P)$ is a logarithmic potential function*

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[1] Numbers in brackets refer to the bibliography at the end of the paper.