THE NONEXISTENCE OF CERTAIN IDENTITIES IN THE THEORY OF PARTITIONS AND COMPOSITIONS

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1. **Introduction.** Let $q_{d,m}(n)$ be the number of partitions of n into parts differing by at least d, each part being greater than or equal to m. We discuss here the question of the existence of identities involving $q_{d,m}(n)$ analogous to the tautology:

$$\sum_{n=0}^{\infty} q_{1,1}(n)x^n = \prod_{\nu=1}^{\infty} (1 + x^\nu),$$

the Euler identity:

$$\sum_{n=0}^{\infty} q_{1,1}(n)x^n = \prod_{\nu=1}^{\infty} \frac{1}{1 - x^{2\nu-1}},$$

and the Rogers-Ramanujan identities:

$$\sum_{n=0}^{\infty} q_{2,1}(n)x^n = \prod_{\nu=0}^{\infty} \frac{1}{(1 - x^{5\nu+1})(1 - x^{5\nu+4})},$$

$$\sum_{n=0}^{\infty} q_{2,2}(n)x^n = \prod_{\nu=0}^{\infty} \frac{1}{(1 - x^{5\nu+2})(1 - x^{5\nu+3})}.$$

We shall in fact show that, aside from the following simple extensions of the first two:

$$\sum_{n=0}^{\infty} q_{1,m}(n)x^n = \prod_{\nu=m}^{\infty} (1 + x^\nu),$$

$$\sum_{n=0}^{\infty} q_{1,m}(n)x^n = \frac{\prod_{\nu=m}^{\infty} (1 - x^{2\nu})}{\prod_{\nu=m}^{\infty} (1 - x^\nu)}$$

$$= \frac{1}{(1 - x^m)(1 - x^{m+1}) \cdots (1 - x^{2m-1})} \prod_{\nu=m}^{\infty} \frac{1}{1 - x^{2\nu+1}},$$

no other such identities exist. More specifically we shall prove the following two theorems, both of which were proved for the case

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