BOOK REVIEWS

Die Bewegungsgruppen der Kristallographie. By J. J. Burckhardt. (Lehrbücher und Monographien aus dem Gebiete der exakten Wissenschaften.) Basel, Birkhauser, 1947. 186 pp. 24.50 S. fr.; bound 29 S. fr.

The systematic study of the discrete space groups and their application to crystallography dates back to Schoenflies (1891) and Federow (1892). Bieberbach in 1910 and Frobenius, with a simpler proof, in 1911 showed that there exist only a finite number of these groups having a finite fundamental region. Treatises listing the 230 space groups and the corresponding point lattices have been written by P. Niggli, *Geometrische Kristallographie des Diskontinuums* (Leipzig, 1919) and R. W. G. Wyckoff, *The analytical expression of the results of the theory of space groups* (Carnegie Institution of Washington Publication 318, 2d ed., 1930). Other contributions to the theory of space groups have been made by D. Hilbert, C. Jordan, L. Schläfli, H. S. M. Coxeter, W. Nowacki, A. Speiser, G. Wintgen, and others to whom references are given. The purpose of the author is to develop the theory of space groups systematically with primary emphasis not on the geometric crystal classes but on the arithmetic theory of space lattices.

Chapter I opens with a careful presentation of the fundamental notions of vectors and matrices, and the particular properties of certain orthogonal and integral unimodular matrices which represent the symmetry operations of a point lattice. The point groups are orthogonal in rectangular coordinates, but integral unimodular when referred to lattice coordinates. If y, x, a are ν-dimensional column vectors, A is a ν by ν matrix and E the unit matrix, then the general rigid motion $y=Ax+a$ is denoted by (A, a). In any of the 230 space groups G, the transformations (E, a) form the invariant abelian subgroup T of translations, and the quotient group G/T is isomorphic to a finite group G_0 of integral unimodular matrices, called a crystal class.

Chapter II contains a description and classification of the crystal classes—the symmetry groups of a given point lattice which leave fixed one lattice point. Two crystal classes are considered *geometrically* equivalent if one can be transformed into the other by a *non-singular* transformation. They are *arithmetically* equivalent only if a transforming matrix can be chosen which is *integral* and *unimodular.*