BOOK REVIEWS

Advanced calculus. By D. V. Widder. New York, Prentice-Hall, 1947. 16+432 pp. $5.00.

This text gives careful treatments of a restricted selection of topics culminating in the theory of real Laplace transforms.

Content: Limits and derivatives of functions of one variable are sketched in four pages. The treatment of partial derivatives emphasizes the theory of functional relations. The differential geometry of curves and surfaces is developed after a short chapter on vectors. A thorough treatment is given to maxima and minima. Single integrals are developed in the form of Stieltjes integrals, with Riemann integrals merely mentioned as a special case. But multiple integrals are given a relatively conventional treatment including some of the usual elementary applications. The theory of line and surface integrals is carried as far as the basic integral relations. Indeterminate forms are handled rather conventionally. The chapters on infinite series and improper integrals are closely parallel. They begin with a relatively detailed treatment of the elementary convergence tests; then follow the theory and applications of uniform convergence; the discussion closes with an introduction to the process of summation. Brief treatments are given to the gamma and beta functions, and to Stirling's formula. Various topics are included in the theory of Fourier series and integrals. The last two chapters develop the theory of real Laplace transforms and their applications to the solution of differential and difference equations. Such subjects as complex variables, matrices, variations, numerical methods, and statistics are omitted entirely; differential equations are only touched on in connection with line integrals and Laplace transforms.

Style: The formulation of most of the definitions and theorems is illustrated by the first numbered definition in the book:

DEFINITION 1. $f(x) \in C$ *at* $x=a \leftrightarrow \lim_{x\to a} f(x)=f(a)$.

Precision and rigor constitute an outstanding feature of this text. The extent to which these attributes are carried is illustrated by the following definition (page 30):

DEFINITION 8. *The directional derivative of* $f(x, y)$ *in the direction* ξ_α *at* (a, b) *is*

$$\left.\frac{\partial f}{\partial \xi_\alpha}\right|_{(a,b)} = \lim_{\Delta s\to 0} \frac{f(a+\Delta s\cos\alpha,\, b+\Delta s\sin\alpha) - f(a, b)}{\Delta s}.$$