NONLINEAR NETWORKS. IIa

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A network is a collection of conducting wires and batteries arbitrarily interconnected. Kirchhoff [1, 6][1] gave a topological-type proof that the currents in the wires are uniquely determined for wires obeying Ohm's law. (Ohm's law is a linear law stating that current and potential drop are proportional.) If the wires obey a nonlinear law, more than one distribution of current is in general possible. For some engineering application a multiplicity of states is desirable as, for example, in counting circuits and oscillators. For other applications it is essential that only one state be possible. It is seldom intuitively evident, however, whether or not a given nonlinear network will have multiple states. Hence, it appears that a qualitative mathematical treatment of nonlinear networks should be of some practical importance [5].

A large class of conductors used in engineering are such that the current through the conductor and the potential drop across the conductor are nondecreasing functions of one another. Such conductors we shall term *quasi-linear*. Examples are: selenium, copper oxide, silicon carbide (thyrite), and thermionic rectifiers [9]. The main result of this note is the proof that *a network of quasi-linear conductors has a stable state of currents, and this state is unique.*

A stable state of currents in a network must satisfy Kirchhoff's laws, which simply are statements of the conservation of electricity and the single valuedness of the potential function. Maxwell [8] discovered two concise ways of expressing these laws: the junction equations and the mesh equations. More or less as a digression we shall show that the mesh equations may be put in the same functional form as the junction equations if and only if the network is planar.

The formulation of mechanical analogs to electric networks has received considerable attention in the literature because of the transfer of techniques suggested by the analogy. We discuss here a different type of analog which we call an *elastic network*. An elastic network is a collection of springs connected to each other at junction points. Forces are applied to the junction points to hold the network in a stretched condition. A tennis net is an example. Electric networks are analogous to one-dimensional elastic networks. Planar or spatial

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[1] Numbers in brackets refer to the references cited at the end of the paper.