

## ON PRINCIPAL LINES AND PRINCIPAL POINTS

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Recently Chang<sup>1</sup> has enriched the contact theory of two plane curves by virtue of many projective correspondences. The object of this paper is to establish some analogous results regarding the contact theory of two space curves.

In an appendix to Fubini and Čech's book<sup>2</sup> Bompiani remarked that the contact theory of two space curves can be treated by considering a double ratio analogous to that of C. Segre.<sup>3</sup> Accordingly Palozzi<sup>4</sup> has obtained principal lines and principal points associated with two space curves which are tangent but have distinct osculating planes or possess a contact of higher order at an ordinary point. But in the latter case Stouffer<sup>5</sup> has shown that neither principal lines nor principal points exist when the principal plane becomes the common osculating plane of the two curves. Naturally in Stouffer's case we should require new covariant figures. We solve this problem by obtaining a new covariant line.

Between the points  $P$  and  $\bar{P}$  of the two space curves  $C$  and  $\bar{C}$ , which are tangent or intersect at an ordinary point, we define a correspondence  $G$ , such that the tangents to  $C$  and  $\bar{C}$  at corresponding points of  $G$  intersect. This correspondence plays an important role in this paper because the joins of the corresponding points of  $G$  constitute developables. We can deduce from  $G$  new definitions of Bompiani's<sup>6</sup> principal lines and principal points for two intersecting space curves.

Two space curves  $C$  and  $\bar{C}$  intersecting at an ordinary point  $O(0, 0, 0)$  with distinct osculating planes may be represented by the

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<sup>1</sup> Chang, *A new definition of Bompiani line and its generalization*, in the hands of The International Cultural Service of China.

<sup>2</sup> Fubini-Čech, *Geometria proiettiva Differenziale*, vol. 2, 1926.

<sup>3</sup> C. Segre, *Sugli elementi curvilinea che hanno come la tangente e il piano osculatore*, Rendiconti della Classe di Scienze Fisiche, Matematiche e Naturali, R. Accademia Nazionale dei Lincei (5) vol. 33 (1924) pp. 325-329.

<sup>4</sup> Palozzi, *Sugli invarianti proiettivi di contratto fra curve sghembe*, Rendiconti della Classe di Scienze Fisiche, Matematiche e Naturali, R. Accademia Nazionale dei Lincei (6) vol. 7 (1928) pp. 321-325.

<sup>5</sup> Stouffer, *On the contact of two space curves*, Bull. Amer. Math. Soc. vol. 38 (1932) pp. 415-419.

<sup>6</sup> Bompiani, *Invarianti d'intersezione di due curve sghembe*, Rendiconti della Classe di Scienze Fisiche Matematiche e Naturali, R. Accademia Nazionale dei Lincei (6) vol. 11 (1931) pp. 456-461.