

rect sum of simple submodules. The real difficulties are met in the attempt to enumerate the two-sided ideals which are contained in the radical.

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ON A CONJECTURE ABOUT INFINITE CLASS FIELDS

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If we are given any algebraic extension field, of finite degree, of a given ground field, then the p -adic completion of the extension field, under any one of its valuations¹ (prime spots) is an algebraic extension of the completion of the ground field under the same valuation. Our original extension field (in the large) thus determines a set of algebraic extensions of p -adic ground fields. We shall refer to these extensions as the *local components* of the original field. If our extension field (in the large) is normal, then any two valuations of the extension field which induce the same valuation in the ground field determine isomorphic local components; hence in case of a normal extension field we can think of a local component as determined by a valuation of the ground field.

When our extension field is not of finite degree we must modify this definition, since the p -adic closure of such an extension field will in general not be algebraic over the ground field.² For a normal extension of infinite degree we define the local component as follows: The original extension is the splitting field³ of a certain set of polynomials with coefficients in the ground field. Define the local component to be the splitting field of this same set of polynomials over the p -adic extension of the ground field. It is easy to show that this field is independent of the set of polynomials used (indeed, one could use the set of all polynomials of the ground field which split in the extension

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¹ For theory of valuations, see E. Artin and G. Whaples, *Axiomatic characterization of fields by the product formula for valuations*, Bull. Amer. Math. Soc. vol. 51 (1945) pp. 469–492, and the literature cited there.

² See Ostrowski, *Über einige Fragen der allgemeine Körpertheorie*, Journal für Mathematik vol. 143 (1914) pp. 225–284. I am indebted to the referee for a correction of the first version of this paper and for this reference.

³ The *splitting field* is the smallest subfield of the algebraic closure in which all the given polynomials split into linear factors.