ON A GENERALIZATION OF THE STIELTJES MOMENT PROBLEM

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The "generalised moment problem"

$$(1)\qquad \int_0^\infty t^{\lambda_n} d\alpha(t) = \mu_n \qquad (0 = \lambda_0 < \lambda_1 < \lambda_2 \cdots < \lambda_n \to \infty)$$

is said to be determined if there is at most one increasing function $\alpha(t)$ satisfying (1) and normalized by $\alpha(0)=0$. R. P. Boas, Jr., who first considered this problem [1][1] gave conditions under which (1) is determined. These do not include the best known result in the classical case $\lambda_n = n$, namely Carleman's criterion: If $\lambda_n = n$ and $\sum \mu_n^{-1/2n} = \infty$, then (1) is determined. I shall now prove a theorem including Carleman's test as a special case. On the other hand this theorem will not include the results of Boas, as I shall from now on assume

$$(2)\qquad \lambda_{n+1} - \lambda_n > c \qquad (n = 1, 2, \cdots)$$

for some $c>0$.

Let

$$\psi(r) = \exp\left\{\sum_{0<\lambda_\nu\leq r} \lambda_\nu^{-1}\right\}.$$

THEOREM. *If there are a non-increasing function $\phi(r)$ and positive constants A and a such that*

$$\psi(r) > A(r/\phi(r))^a$$

and if

$$(3)\qquad \sum_2^\infty \frac{\lambda_n - \lambda_{n-1}}{\mu_n^{1/a\lambda_n}\phi(\lambda_{n-1})} = \infty,$$

then (1) *is determined.*

The proof is based on the following lemma.

LEMMA. *If* (2) *is the case, then*

$$G(z) = \prod_{\nu=1}^\infty \frac{\lambda_\nu + z}{\lambda_\nu - z} e^{-2z/\lambda}$$

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[1] Numbers in brackets refer to the references cited at the end of the paper.