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UNIVERSITY OF CALIFORNIA AT LOS ANGELES

THE SPACE L^ω AND CONVEX TOPOLOGICAL RINGS

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1. **Introduction.** The motive for investigating the class L^ω of functions belonging to all L^p-classes has no measure-theoretic origin: it was our desire to discover whether or not in every convex metric ring[1] R one could find a system $\{U\}$ of convex neighborhoods of 0 having the property that $f, g \in U$ implies $fg \in U$. We show here that L^ω has no proper convex open set U containing 0 and satisfying the relation $UU \subset U$, thus supplying the desired counter-example.

The significance of neighborhood systems of the type $\{U\}$ described above is made somewhat clearer by a proof that they insure the existence and continuity of entire functions (for example, the exponential function) on the topological ring R.

Such neighborhood systems $\{U\}$ are always present in rings of continuous real-valued functions over any space, provided that convergence means uniform convergence on compact sets.

We also consider the relation of L^∞, L^ω, and the L^p-classes, since L^ω does not seem ever to have been discussed as a topological and algebraic entity.

2. **Notation and elementary facts.** Let us consider measurable functions defined on $[0, 1]$. For $p \geqq 1$ we shall consistently employ the usual notation

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[1] More precisely, metrizable, convex, complete topological linear algebra. For these one requires continuity in both ring operations and scalar multiplication. It will appear that L^ω has these properties.