EQUIVALENCE IN A CLASS OF DIVISION ALGEBRAS OF ORDER 16

R. D. SCHAFER

Let $\mathfrak{C}$ be a Cayley-Dickson division algebra over an arbitrary field $\mathfrak{F}$ with principal equation

$$x^2 - t(x)x + n(x) = 0 \tag{1}$$

and involution

$$S\colon \quad x \leftrightarrow xS = t(x) - x. \tag{2}$$

We are concerned with division algebras $\mathfrak{A}$ of order 16 over $\mathfrak{F}$ defined in the following way: let $\mathfrak{C}_o$ be a division algebra (of order 8) over $\mathfrak{F}$ with the same elements as $\mathfrak{C}$ but with multiplication denoted by xoy; further let $\mathfrak{A} = \mathfrak{C} + v\mathfrak{C}$, multiplication[1] in $\mathfrak{A}$ being defined by

$$cz = (a + vb)(x + vy) = (ax + yobS) + v(aS\cdot y + xb) \tag{3}$$

for a, b, x y, in $\mathfrak{C}$.

In the original form of this paper, the author considered the problem of equivalence in the class of algebras $\mathfrak{A} = \mathfrak{C} + v\mathfrak{C}$ with multiplication defined by

$$cz = (a + vb)(x + vy) = (ax + g\cdot ybS) + v(aS\cdot y + xb) \tag{4}$$

for a, b, x, y in $\mathfrak{C}$ where g is a fixed element of $\mathfrak{C}$, $g \notin \mathfrak{F}$. The author had shown in [5] that $\mathfrak{A}$ is a division algebra in case g is chosen with $n(g)$ not a square in $\mathfrak{F}$; in particular, such a choice of g can be made when $\mathfrak{F}$ is the field R of rational numbers. R. H. Bruck, the referee of the paper in its original form, suggested a study of the wider class of algebras defined by (3). Theorems 1 and 2 are generalizations of the result in [5] and are due[2] to R. H. Bruck. By their use the class of algebras studied here has been considerably enlarged.[3]

In §2 we shall determine conditions for the equivalence of two alge-

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[1] This modification of the Cayley-Dickson process was originally presented by R. H. Bruck in [2], Theorem 16C, to obtain non-alternative division algebras of orders 4 and 8. Numbers in brackers refer to the references cited at the end of the paper.

[2] Theorem 2 was communicated to the author by Bruck, complete except for the proof of the equivalence of $\mathfrak{C}_*$ and $\mathfrak{C}$, a fact which Bruck conjectured.

[3] See the comment following the corollary to Theorem 4.