

$w = (N-1)^{1/2} s / \chi_{1-\alpha}$. It is shown that if D be defined as follows: (1) if $L_1 \leq \bar{x} \leq L_2$, $D = (2\pi)^{-1/2} \int_{(L_1 - \bar{x})/w}^{(L_2 - \bar{x})/w} \exp \{ -y^2/2 \} dy$, (2) $D = G$ otherwise, then $|P\{D \leq \gamma\} - \alpha|$ approaches zero as $N \rightarrow \infty$. Thus D is a large sample lower confidence limit. The extension to upper and two-sided limits presents no difficulty. (Received July 8, 1946.)

TOPOLOGY

336. R. F. Arens: *Convex topological algebras*. Preliminary report.

A convex topological algebra A is a convex topological linear space in which a multiplication of elements is defined, which is, as is addition and scalar multiplication (for definiteness, take the case of real scalars), continuous simultaneously in both factors. This is a generalization of the concept of normed rings. However, the elements with inverses do not form an open set, nor is inversion continuous when possible. The author proves that if A is a division algebra, and is complete in some metric, then A is finite-dimensional, and hence its structure follows from Frobenius' theorem. This result is fundamental for the representation theory of convex topological algebras. (Received June 7, 1946.)

337. R. F. Arens: *Duality in topological linear spaces*.

Let L be any topological linear space with elements x . Let L^* be the set of continuous linear functionals f defined on L , and use in L^* the topology in which convergence of directed sets means uniform convergence on each compact subset of L (the k -topology). Let this construction be repeated, using L^* instead of L , and giving rise to L^{**} , with elements X . Then for each X there is an $x \in L$ such that $X(f) = f(x)$ for each $f \in L^*$. This natural mapping from L^{**} back into L is 1-1 and continuous if L is convex; if furthermore L is complete in some invariant metric (in particular, if L is a Banach space) then the natural mapping is bicontinuous. (Received July 10, 1946.)

338. R. H. Bing: *Skew sets*.

No plane set G contains a collection of five mutually separated sets such that the closure of the sum of any pair of these five sets is the closure of a connected subset of G which is open in G . (Received July 10, 1946.)

339. G. D. Birkhoff and D. C. Lewis: *Chromatic polynomials*.

The number of ways a map P_{n+3} of $n+3$ regions can be colored with λ colors is given by a polynomial $P_{n+3}(\lambda)$ of degree $n+3$. Certain new properties of these chromatic polynomials are established. For instance, if P_{n+3} is regular and if $(-1)^k a_k$ is the coefficient of $(\lambda-2)^{n-k}$ in the expansion of $P_{n+3}(\lambda)/\lambda(\lambda-1)(\lambda-2)$ in powers of $\lambda-2$, it is shown that binomial coefficient $C_n^k \leq a_k \leq C_n^{n+k-1}$. Similar results are obtained for expansions in powers of $\lambda-5$. Moreover, extensive numerical calculations indicate that both $P_{n+3}(\lambda)/\lambda(\lambda-1)(\lambda-2) - (\lambda-3)^n$ and $(\lambda-2)^n - P_{n+3}(\lambda)/\lambda(\lambda-1)(\lambda-2)$ are positively completely monotonic for $\lambda \geq 4$. This conjecture is a very strong form of the usual four-color proposition that $P_n(4) > 0$. In connection with reducibility, reduction formulas, and the analysis of rings, the theory of Kempe chains, which has been applied qualitatively with considerable success to the case $\lambda=4$, is generalized so as to yield quantitative results on chromatic polynomials for all values of λ . Typical results on reducible configurations, previously obtained only by use of Kempe chains, are also obtained inductively. The present paper therefore to some extent attempts to