WATER WAVES ON SLOPING BEACHES

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1. **Introduction.** Consider the time-dependent velocity potential $\phi(x, y, t)$ of the motion of an incompressible inviscid liquid in two space dimensions x, y. The Newtonian equations and the condition of incompressibility lead to the following mathematical model: Laplace's equation

$$\frac{\partial^2\phi}{\partial x^2} + \frac{\partial^2\phi}{\partial y^2} = 0 \tag{1.1}$$

and Bernoulli's law

$$\frac{\partial\phi}{\partial t} + \left(\left(\frac{\partial\phi}{\partial x}\right)^2 + \left(\frac{\partial\phi}{\partial t}\right)^2\right)\Big/ 2 + gy + p = \text{const.}, \tag{1.2}$$

relating the pressure p and the gravity potential gy per unit mass to the velocity potential; here the x-axis is horizontal, the y-axis vertical and upwards and g is the modulus of gravity acceleration. At the boundary of air and water the pressure is supposed to have a constant value p_0; thus (1.2) relates implicitly the surface elevation y to the velocity potential. A considerable simplification is introduced by assuming the motion to be small of first order so that in (1.2) the quadratic terms may be cancelled. Then the motion becomes a small perturbation of the equilibrium position in which the surface will be thought of as given by $y=0$. For small elevations of the surface one concludes from (1.2) that, but for terms of higher order,

$$y = y(x, t) = -\frac{\partial\phi}{\partial t}(x, 0, t)\Big/ g, \tag{1.3}$$

where the constant on the right hand has been absorbed in a properly modified ϕ. To this equation is added the condition that a particle at the surface remains at the surface; that is (using the "substantial" time derivative of (1.2)),

$$dy(x, t)/dt = -\frac{\partial^2\phi}{\partial t^2}\Big/ g = \partial\phi/\partial y, \tag{1.3.1}$$

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