ON ANALYTIC FUNCTIONS WITH BOUNDED CHARACTERISTIC

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A function $f(re^{i\phi})$, regular within the unit circle, is called a function with bounded characteristic if

$$\lim_{r\to 1}\int_0^{2\pi}\log^+\left|f(re^{i\phi})\right|d\phi$$

is bounded, where $\log^+\left|f(re^{i\phi})\right| = \max\left(\log\left|f(re^{i\phi})\right|, 0\right)$. If $f(z)$ is a function with bounded characteristic, then

$$\lim_{r\to 1} f(re^{i\phi}) = f(e^{i\phi})$$

exists almost everywhere [1].[2]

In the first part of this paper we prove the following:

THEOREM I. *Let* $\{f_n(z)\}$ $(n=1, 2, 3, \cdots)$ *and* $f(z)$ *be functions with bounded characteristics, let*

$$(1)\qquad \begin{aligned} \log A_n &= \lim_{r\to 1}\int_0^{2\pi}\log^+\left|f_n(re^{i\phi})\right|d\phi, \\ \log A &= \lim_{r\to 1}\int_0^{2\pi}\log^+\left|f(re^{i\phi})\right|d\phi, \end{aligned}$$

and

(2) $\left|f(e^{i\phi})-f_n(e^{i\phi})\right| < m_n$, *for* $\phi\in E_n$, *and let* μ_n *be the measure of* E_n.

If

$$(3)\qquad \lim_{n\to\infty} m_n^{\mu_n} = 0,$$

and for every positive σ *there exists a positive integer* n_σ *such that*

$$(4)\qquad A_n < m_n^{-\sigma\mu_n} \quad \text{for} \quad n > n_\sigma,$$

then the sequence $\{f_n(z)\}$ *tends uniformly to* $f(z)$ *in any closed domain interior to the unit circle.*

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[1] Deceased December 23, 1945.

[2] Numbers in brackets refer to the Bibliography at the end of the paper.