BOUNDED J-FRACTIONS

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1. Introduction. A J-fraction

$$\frac{1}{b_1 + z - \cfrac{a_1^2}{b_2 + z - \cfrac{a_2^2}{b_3 + z - \cdots}}}, \tag{1.1}$$

in which the coefficients a_p and b_p are complex constants and z is a complex parameter, is said to be *bounded* if there exists a constant M such that

$$|a_p| \leqq M/3, \qquad |b_p| \leqq M/3, \qquad p = 1, 2, 3, \cdots. \tag{1.2}$$

This condition can be formulated in terms of J-forms in accordance with the following theorem.

THEOREM 1.1. *The J-fraction* (1.1) *is bounded if and only if there exists a constant N such that*

$$\left| \sum_{p=1}^{n} b_p u_p v_p - \sum_{p=1}^{n-1} a_p(u_p v_{p+1} + u_{p+1} v_p) \right| \leqq N \left(\sum_{p=1}^{n} |u_p|^2 \cdot \sum_{p=1}^{n} |v_p|^2 \right)^{1/2}, \qquad n = 1, 2, 3, \cdots, \tag{1.3}$$

for all values of the variables u_p and v_p, the constant N being independent of the variables and of n.

In fact, if (1.3) holds then we find, on specializing the values of the u_p and v_p, that $|b_p| \leqq N$, $|a_p| \leqq N$, $p = 1, 2, 3, \cdots$; and if (1.2) holds then, by Schwarz's inequality, (1.3) holds with $N = M$.

If (1.3) holds, then the J-form $\sum b_p u_p v_p - \sum a_p(u_p v_{p+1} + u_{p+1} v_p)$ is said to be *bounded*, and the least value of N which can be used in that inequality is called the *norm* of the J-form. We shall also call this number the *norm* of the J-fraction. When (1.2) holds then, as pointed out above, (1.3) holds with $N = M$. *Hence the norm of the J-fraction does not exceed the least number M which can be used in* (1.2).

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