

BIBLIOGRAPHY

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A REMARK ON DENSITY CHARACTERS

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Let X be an arbitrary topological space satisfying the T_0 -separation axiom [1, Chap. 1, §4, p. 58].² We recall the following definition [3, p. 329].

DEFINITION 1. *The least cardinal number of a dense subset of the space X is said to be the density character of X . It is denoted by the symbol $\mathfrak{X}(X)$.*

We denote the cardinal number of a set A by $|A|$.

Pospišil has pointed out [4] that if X is a Hausdorff space, then

$$(1) \quad |X| \leq 2^{2^{\mathfrak{X}(X)}}.$$

This inequality is easily established. Let D be a dense subset of the Hausdorff space X such that $|D| = \mathfrak{X}(X)$. For an arbitrary point $p \in X$ and an arbitrary complete neighborhood system \mathcal{U}_p at p , let \mathcal{D}_p be the family of all sets $U \cap D$, where $U \in \mathcal{U}_p$. Thus to every point of X , a certain family of subsets of D is assigned. Since X is a Hausdorff space, $\mathcal{D}_p \neq \mathcal{D}_q$ whenever $p \neq q$, and the correspondence assigning each point p to the family \mathcal{D}_p is one-to-one. Since X is in one-to-one correspondence with a sub-hierarchy of the hierarchy of all families of subsets of D , the inequality (1) follows.

It may be remarked in passing that the inequality (1) does not obtain for all T_1 -spaces. Let m be a cardinal number greater than 2^c , where $c = 2^{\aleph_0}$. Let Z be a T_1 -space of cardinal number m and with the property that the only closed proper subsets of Z are finite or

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² Numbers in brackets refer to the Bibliography at the end of the paper.