A GEOMETRICAL CHARACTERIZATION FOR THE AFFINE DIFFERENTIAL INVARIANTS OF A SPACE CURVE

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1. **Introduction.** Let $x = x(s)$ be the vector equation of a space curve C with the affine arc length s as parameter. It is known that $x(s)$ satisfies a differential equation of the following form [1, p. 73; 3, p. 76][1]

$$x'''' + kx'' + tx' = 0, \tag{1.1}$$

where the primes represent derivatives with respect to s. The vector x' is the *tangent vector* and the vectors x'' and x''' are called the *affine principal normal* and the *affine binormal*, respectively, of the curve C at the point considered. The vectors x', x'', x''' with the initial point at the point x of the curve C constitute the *affine fundamental trihedral* at x and they satisfy the following relation [1, p. 72; 3, p. 78]

$$(x', x'', x''') = 1. \tag{1.2}$$

The plane determined by the point x and the edges x', x'' of the affine fundamental trihedral is the *osculating plane* at x; the plane determined by x and the edges x'', x''' is the *affine normal plane* and the plane determined by x and the edges x', x''' is the *affine rectifying plane* of the curve C at the point x.

Sometimes it is convenient to use the vector $kx'/4 + x'''$ which is called the *binormal of Winternitz* [1, p. 76]. The invariants k and t (functions of the affine arc length s) are called the *affine curvature* and the *affine torsion* respectively.

For the affine fundamental trihedral and for k and t some geometrical characterizations have been given by Blaschke [1, chap. 3], Salkowski [3, p. 76] and Haack [2]. The purpose of the present paper is to give a new geometrical construction for the mentioned elements, which we believe is simpler than those previously obtained.

2. **Geometrical elements associated to an ordinary point of a space curve.** Let us consider the space curve C represented by the vector equation $x = x(s)$ (s = affine arc length) in the neighborhood of the point $x_0 = x(0)$. If we denote by $x_0^{(i)}$ the derivatives $d^{(i)}x/ds^i$ at the point $s = 0$, since x_0', x_0'', x_0''' are not coplanar (by (1.2)), any point y of the space can be expressed in the form

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[1] Numbers in brackets refer to the references cited at the end of the paper.