then the sequence $f(nt)$ is complete in $L_\infty(0, \pi)$. (3) Let $f(t) = t - [t] - 1/2, f(n) = 0$; the sequences $\{f(nt)\}$ and $\{\sin 2n\pi t\}$ have the same span in $C(0, 1/2)$. (4) $f(t) = \text{sgn} \sin n\pi t$; the sequence $\{f(nt)\}$ is complete in $L_r(0, 1)$ for all $r > 1$. (5) Each of the sequences $1, e^{-t}, t/(e^{nt}-1); 1, e^{-t}, t^2/(e^{nt}-1)^2, n = 1, 2, 3, \cdots$, is complete in $C(0, \infty)$. (Received May 31, 1946.)

241. Hing Tong: *Ideals of normed rings associated with topological spaces*. Preliminary report.

Let R be a perfectly normal bicompact space, $\mathfrak{R}$ the normed abelian ring of complex-valued functions continuous over R (for $f \in \mathfrak{R}$, $\|f\| = \max_R |f(x)|$), $\mathfrak{J}$ any (closed) ideal in $\mathfrak{R}$. Then $\mathfrak{J}$ is a principal ideal. If $\prod_{f \in \mathfrak{J}} \mathfrak{N}_f = N_{\mathfrak{J}}$ ($\mathfrak{N}_f$ denotes the set of zeros of f) corresponds to $\mathfrak{J}$, the correspondence is an isomorphism (that is, $\mathfrak{J} \leftrightarrow N_{\mathfrak{J}}$) such that ($\alpha_1$) $\mathfrak{J}_1 \cup \mathfrak{J}_2 \leftrightarrow N_{\mathfrak{J}_1} \cdot N_{\mathfrak{J}_2}$, ($\alpha_2$) $\mathfrak{J}_1 \cap \mathfrak{J}_2 \leftrightarrow N_{\mathfrak{J}_1} + N_{\mathfrak{J}_2}$, ($\alpha_3$) $\mathfrak{R} \leftrightarrow \{0\}$, ($\alpha_4$) $\sum_\alpha \mathfrak{J}_\alpha \leftrightarrow \prod_\alpha N_{\mathfrak{J}_\alpha}$, ($\alpha_5$) $\prod_\alpha \mathfrak{J}_\alpha \leftrightarrow$ closure of $\sum_\alpha N_{\mathfrak{J}_\alpha}$. Conversely, let $\mathfrak{R}$ be the normed ring of bounded complex-valued functions continuous over a topological space R. The conditions $\mathfrak{J} \leftrightarrow N_{\mathfrak{J}}$, ($\alpha_1$) holds, and every $\mathfrak{J} \subseteq \mathfrak{R}$ is principal imply that R is a perfectly normal bicompact space. If R is a T_1' (a space $\ni y \in \bar{x} \rightarrow \bar{x} = \bar{y}$) bicompact normal space, the above results hold providing "principal ideal" is replaced by "ideal." The results also hold for bicompact normal spaces if isomorphism is relaxed to homomorphism. The ring of continuous mappings (continuous in the strong topology) of R into the ring of bounded linear operators over a Banach space with a basis leads to the same results if an ideal means a two-sided ideal. The results do not hold for non-separable Banach spaces. (Received April 10, 1946.)

Applied Mathematics

242. Garrett Birkhoff: *Symmetric Lagrangian systems*.

Let Ω be any Lagrangian dynamical system with no potential energy and kinetic energy function $L = 2^{-1} E_{jk} \dot{q}_j \dot{q}_k$. Suppose that L is invariant under a simply transitive group G of rigid motions on its configuration space. Then the "generalized force" required to maintain motion along a one-parameter subgroup in the jth coordinate direction has the components $Q_i = c^{ij}_{\ k} E_{jk}$, where the $c^{ij}_{\ k}$ are the structure constants of G. It is a corollary that the d'Alembert paradox would take in non-Euclidean geometry the following form. A rigid body moving under translation, rotation, or screw motion along an axis in an incompressible, nonviscous fluid without circulation will experience no thrust along or torque about the axis. However cross-force is possible. (Received May 13, 1946.)

243. R. J. Duffin: *Nonlinear networks*. III.

A system of n nonlinear differential equations is shown to have a periodic solution. The interest of these equations is that they describe the vibrations of electrical networks under a periodic impressed force. Consider an arbitrary linear network of inductors, resistors and capacitors which does have a solution for a given periodic impressed force. The main result of this note states that the existence of a periodic solution is still guaranteed if the linear resistors of such a network are replaced by *quasi-linear* resistors. A quasi-linear resistor is one whose differential resistance lies between positive limits. No other sort of nonlinearity besides this type of nonlinear damping is considered. The proof rests on the closure properties of nonlinear transformations in Hilbert space. (Received May 7, 1946.)