*where $C$ is an arbitrary analytic Jordan curve, $z=\alpha$ is a point interior to $C$, $f(z)$ is of class $E_p$ interior to $C$, and $n(z)$ is the modulus on $C$ of a function $N(z)$ analytic and nonvanishing in the closed region $\Gamma$, is*

$$F_0(z) = A\left[\frac{N(\alpha)}{N(z)}\cdot\frac{g'(z)}{g'(\alpha)}\right]^{1/p}.$$

*Let $P_n(z)$ be the corresponding minimizing polynomial of degree $n$. Then the sequence $P_n(z)$, $n=0, 1, 2, \cdots$, converges maximally to $F_0(z)$ on $\Gamma$.*

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# NOTE ON THE LOCATION OF THE CRITICAL POINTS OF HARMONIC FUNCTIONS

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The object of this note is to publish the statement of the following theorem.

THEOREM I. *In the extended $(x, y)$-plane let $R_0$ be a simply-connected region bounded by a continuum $C_0$ not a single point, and let the disjoint continua $C_1, C_2, \cdots, C_n$ lie interior to $R_0$ and together with $C_0$ bound a subregion $R$ of $R_0$. By means of a conformal map of $R_0$ onto the unit circle we define in $R_0$ non-euclidean lines, the images of arbitrary circles orthogonal to the unit circle. Denote by $\Pi$ the smallest closed non-euclidean convex region in $R_0$ which contains $C_1, C_2, \cdots, C_n$.*

*Let the function $u(x, y)$ be harmonic interior to $R$, continuous in the closure of $R$, with the values zero on $C_0$ and unity on $C_1, C_2, \cdots, C_n$. Then the critical points of $u(x, y)$ in $R$ are $n-1$ in number and lie in $\Pi$.*

Critical points are of course to be counted according to their multiplicities.

A limiting case of Theorem I has already been established:[1] if $f(z)$

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[1] J. L. Walsh, Bull. Amer. Math. Soc. vol. 45 (1939) pp. 462–470; see p. 465. The result was proved later by W. Gontcharoff, C. R. (Doklady) Acad. Sci. URSS. vol. 36 (1942) pp. 39–41.