$\rho(r)=\xi(r)$ for $t_2\leqq r\leqq s_2$ where t_2 $(<s_2)$ is the point nearest to s_2 at which $\xi(t_2)=\sigma(t_2)$. If $\xi(s_2)=\sigma(s_2)$, then let $t_2=s_2$. For $r<t_2$ let $\rho(r)=\rho(t_2)+\log\log\log t_2-\log\log\log r$ for $u_1\leqq r\leqq t_2$ where u_1 $(<t_2)$ is the point of intersection of $y=\rho$ with $y=\rho(t_2)+\log\log\log t_2-\log\log\log r$.

Let $\rho(r)=\rho$ for $r_1\leqq r\leqq u_1$. It is always possible to choose r_2 so large that $r_1<u_1$. We repeat the procedure and note that

$$\rho(r) \geqq \xi(r) \geqq \sigma(r)$$

and $\rho(r)=\sigma(r)$ for $r=t_1, t_2, t_3, \cdots$. Hence $\lim_{r\to\infty}\rho(r)=\rho$, and

$$\limsup_{r\to\infty}\frac{\log M(r)}{r^{\rho(r)}} = 1.$$

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A NOTE ON THE SPECTRAL THEOREM

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1. Introduction. Although the connections between the spectral resolution of a self-adjoint transformation in Hilbert space, the moment problem, and Riesz' integral representation [1][1] for linear functionals on the space C are known (cf. Stone [2], Murray [3], Widder [4], Lengyel [5]), the following elementary derivation of the spectral theorem from the Riesz theorem exhibits the connections in, perhaps, the simplest light. We consider only *bounded* self-adjoint transformations H; one can treat an unbounded H by considering $(I+H^2)^{-1}$, which is bounded and self-adjoint [3, p. 95]. Note that the derivation does not involve the separability of the Hilbert space $\mathfrak{H}$.

2. Six lemmas. Let H be a self-adjoint transformation with the bounds a, b—that is, $a\|f\|^2\leqq(Hf, f)\leqq b\|f\|^2$ for all $f\in H$, and $\|H\|=\max(|a|, |b|)$. Denote by C the space of continuous real-valued functions defined on the closed interval (a, b), with $\|f(x)\|=\max|f(x)|$ $(a\leqq x\leqq b)$. Let $p(x)=\sum_0^n c_i x^i$ be any polynomial with real coefficients, and let $p(H)$ be the corresponding transformation $p(H)=\sum_0^n c_i H^i$, where $H^0=I$.

Received by the editors September 6, 1945.

[1] Numbers in brackets refer to the references cited at the end of the paper.