with the unbiased estimate of least variance. Thus the classical estimates of the mean and the variance are justified from a new point of view, and also computable estimates of all higher moments are presented. It is interesting to note that for n greater than 3 neither the sample nth moment about the sample mean nor any constant multiple thereof is an unbiased estimate of the nth moment about the mean. (Received October 6, 1945.)

44. Isaac Opatowski: *Markoff chains and Tchebychev polynomials.*

Let the possible states be $0, 1, \cdots, n+1$ and the only transitions possible during any time dt $(i-1\rightarrow i)$ for $1\leqq i\leqq n+1$ and $(i+1\rightarrow i)$ for $0\leqq i\leqq n-1$. Let the conditional probabilities for these transitions be respectively $k_i dt+o(dt)$ and $g_i(dt)+o(dt)$, where $k_i=k$ for $1\leqq i\leqq n$, $k_{n+1}=k$ or 0, $g_i=g$ or 0, k and g being two positives constants. The probability $P(t)$ of the existence of the state n at a time t if the state 0 existed at $t=0$ is in the general case a convolution of particular functions $P(t)$ corresponding to the following cases: (i) $k_{n+1}=0$, $g_i=g$ $(i\leqq n-1)$; (ii) $k_{n+1}=0$, $g_i=g$ $(i\leqq n-2)$, $g_{n-1}=0$; (iii) $k_{n+1}=k$, $g_i=g$ $(i\leqq n-1)$. In (i), $p(s)=\int_0^\infty e^{-st}P(t)dt=(k/g)^{n/2}/[sU_n(x)]$, where $U_n(x)$ is the Tchebychev polynomial of second kind and x is a linear function of s. The roots of U_n give an explicit expression of $P(t)$ as a linear combination of n exponentials whose coefficients are calculated in a form convenient for computations. In cases (ii) and (iii), $[p(s)]^{-1}$ is a linear combination of two U_i's and the roots of $[p(s)]^{-1}$ are located within narrow ranges, which makes the calculation of $P(t)$ possible within any accuracy desired. These chain processes occur in some biophysical phenomena and the paper will appear in Proc. Nat. Acad. Sci. U.S.A. under a slightly different title. (Received October 11, 1945.)

Topology

45. Lipman Bers and Abe Gelbart: *A remark on the Lebesgue-Sperner covering theorem.*

A new and elementary proof is given of a somewhat stronger form of the well known Lebesgue-Sperner covering theorem (Math. Ann. vol. 70 p. 166; Abh. Math. Sem. Hamburgischen Univ. vol. 6 p. 265). Some corollaries are discussed. (Received October 19, 1945.)

46. R. H. Bing: *Solution of a problem of J. R. Kline.*

It is shown that a locally connected, compact, metric continuum S is topologically equivalent to the surface of a sphere provided no pair of points separates S but every simple closed curve separates S. On the assumption that an arc separates S, a simple closed curve is constructed that does not separate S. (Received October 10, 1945.)

47. O. G. Harrold: *The* ULC *property of certain open sets.* I. *Euclidean domains.*

Let M be a compact continuum which separates Euclidean 3-space. If M is deformation-free into a complementary domain A and $p^1(M)=0$, then the fundamental group of A vanishes. By means of this: if M^* is a compact continuum separating 3-space which is deformation-free into a complementary domain A, then A is ULC. If, in addition, $p^1(M^*)=0$ and A is bounded, this implies $\bar{A}$ is a singular 3-cell by a result of S. Eilenberg and R. L. Wilder (Amer. J. Math. vol. 64 (1942) pp. 613–622).