

**the surface.** The intersector net may be required to coincide with a net other than the lines of curvature. For example, in case the asymptotic net on  $S$  is parametric,  $d_{11} = d_{22} = 0$ , so that if the intersector net is required to coincide with the asymptotic net, equations (25) become

$$(29) \quad (d^{\gamma\alpha}\omega_{,\alpha})_{,\alpha} + g^{\gamma\gamma}d_{\alpha\gamma}\omega = 0 \quad (\gamma \neq \alpha; \alpha, \gamma \text{ not summed}).$$

On putting  $\gamma = 2, \alpha = 1$ , and making use of the Codazzi relation

$$\partial d_{12}/\partial u^1 = d_{12}(\Gamma^1_{11} - \Gamma^2_{12}),$$

there results from equations (29)

$$\partial^2\omega/\partial u^1\partial u^1 + \Gamma^1_{11}\partial\omega/\partial u^1 - \Gamma^2_{11}\partial\omega/\partial u^2 + g^{22}(d_{12})^2\omega = 0,$$

and similarly, with  $\gamma = 1, \alpha = 2$ ,

$$\partial^2\omega/\partial u^2\partial u^2 - \Gamma^1_{22}\partial\omega/\partial u^1 + \Gamma^2_{22}\partial\omega/\partial u^2 + g^{11}(d_{12})^2\omega = 0.$$

*To a solution  $\omega(u^1, u^2)$  of the last two equations there corresponds a congruence  $\Lambda$  for which the developables intersect the surface  $S$  in its asymptotic curves.*

UNIVERSITY OF OKLAHOMA

## INTEGRAL DISTANCES

PAUL ERDÖS

In a note under the same title (Bull. Amer. Math. Soc. vol. 51 (1945) pp. 598–600) it was shown that there does not exist in the plane an infinite set of noncollinear points with all mutual distances integral.

It is possible to give a shorter proof of the following generalization: if  $A, B, C$  are three points not in line and  $k = [\max(AB, BC)]$ , then there are at most  $4(k+1)^2$  points  $P$  such that  $PA - PB$  and  $PB - PC$  are integral. For  $|PA - PB|$  is at most  $AB$  and therefore assumes one of the values  $0, 1, \dots, k$ , that is,  $P$  lies on one of  $k+1$  hyperbolas. Similarly  $P$  lies on one of the  $k+1$  hyperbolas determined by  $B$  and  $C$ . These (distinct) hyperbolas intersect in at most  $4(k+1)^2$  points. An analogous theorem clearly holds for higher dimensions.

UNIVERSITY OF MICHIGAN

Received by the editors September 18, 1945.