GENERALIZATIONS OF TWO THEOREMS OF JANISZEWSKI

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Janiszewski proved [2][1] the following for the plane.

THEOREM A. *The sum of two compact continua does not cut the point A from the point B provided neither cuts A from B and their common part is connected or does not exist.*

THEOREM B. *The sum of two compact continua cuts the plane provided their common part is not connected.*

Many generalizations and modifications of these theorems appear in the literature (for example, see [**1**, **3**, **4**, **6**, **8**, **9**, **10**, **11**, **12**]). This paper generalizes the two theorems by considering, instead of continua, sets that are neither open nor closed. All sets referred to in this paper are in the plane.

DEFINITION. The set R cuts the point A from the point B in the set W provided some continuum in W contains A and B and R intersects neither A nor B but it intersects every continuum in W containing A and B. If R cuts A from B in the plane, we simply say that it cuts A from B. Thus we use *cut* in the sense that *coupe* was used in Fundamenta Mathematicae (for example, see [**1**, p. 75, and **10**, p. 15]) and not in the sense used by some writers to mean separate. If a closed set cuts two points from each other in the plane, it separates them from each other; this is not true for more general sets.

Generalizations of Theorem A. We shall make use of the following known result [**12**, p. 129, and **4**, pp. 35–36].

THEOREM 1. *If neither of the closed point sets H and K separates the point A from the point B, the common part of H and K is connected or does not exist and the part of H in the complement of K is compact, then the sum of H and K does not separate A from B.*

THEOREM 2. *If neither of the point sets H and K separates the point A from the point B, each of the sets is closed in their sum,[2] one of the sets is compact[3] and their common part is a continuum or does not exist, then the sum of H and K does not separate A from B.*

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[1] Numbers in brackets refer to the references cited at the end of the paper.

[2] The set H is closed in the set S if $\bar{H} \cdot S$ is equal to H.

[3] In Theorems 2, 5, 6, 7, 8, 9, 10 instead of assuming that one of the sets is compact, we can assume that the part of it in the complement of the other is compact.