BOUNDS FOR THE CHARACTERISTIC ROOTS OF A MATRIX

EDWARD W. BARANKIN

In a recent paper [1],[1] A. B. Farnell reiterated the possibility of the existence of a certain upper bound for the characteristic roots of a matrix. This bound was first conjectured by W. V. Parker (cf. [1]), and was, in fact, proved in the case of matrices of order 2 by Farnell. We shall here, in Theorem 1 below, establish that bound for matrices of *any* order n. Two previous results on bounds for characteristic roots, one due to Parker [2], and one to Farnell [1], will be stated for purposes of comparison with Theorem 1. But first, we make certain definitions, following, for the most part, those of Farnell, for the sake of uniformity.

We let $A = \|a_{rs}\|$, of order n, be any matrix of complex numbers, with a characteristic root $\lambda = \alpha + i\beta$ different from 0. Then there exists a non-null vector $\{x_r\}$, $r = 1, \cdots, n$, such that

$$\lambda x_r = \sum_s a_{rs} x_s. \tag{1}$$

With A^* denoting the hermitian-conjugate of A, set

$$(A + A^*)/2 = B = \|b_{rs}\|, \qquad (A - A^*)/2i = C = \|c_{rs}\|,$$

both of which matrices are hermitian. Finally, define

$$R_r = \sum_s |a_{rs}|, \qquad T_s = \sum_r |a_{rs}|,$$

$$R = \max_{(r)} R_r, \qquad T = \max_{(s)} T_s,$$

$$S_r' = \sum_s |b_{rs}| = \sum_s |b_{sr}|, \qquad S_r'' = \sum_s |c_{rs}| = \sum_s |c_{sr}|.$$

Now the theorems of Parker and Farnell may be stated briefly as follows:

PARKER'S THEOREM. *If S, S', and S'' are the greatest of the $(R_r + T_r)/2$, S_r', and S_r'', respectively, then*

$$|\lambda| \leqq S, \qquad |\alpha| \leqq S', \qquad |\beta| \leqq S''.$$

FARNELL'S THEOREM.[2] $|\lambda|^2$ *is bounded above by* RT.

Presented to the Society, February 24, 1945; received by the editors March 20, 1945.

[1] Numbers in brackets refer to the references cited at the end of the paper.

[2] This is not the strongest bound that Farnell obtains.