ON SIMPLE GROUPS OF FINITE ORDER. I

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1. Introduction. Using the theory of representations of groups we have obtained a number of results for simple groups of certain types of orders. In the present paper, we shall prove the following result: If $\mathfrak{G}$ is a (non-cyclic) simple group of order $g = pq^b g^*$, where p and q are two primes and where b and g^* are positive integers with $g^* < p-1$, then either $\mathfrak{G} \cong LF(2, p)$[1] with $p = 2^m \pm 1$, $p > 3$, or $\mathfrak{G} \cong LF(2, 2^m)$ with $p = 2^m + 1$, $p > 3$; conversely, these groups satisfy the assumptions. As an application, we determine all simple groups of order prq^b, where p, r, q are primes and where b is a positive integer. The only simple groups of this type are the well known groups of orders 60 and 168.

2. Some known results concerning representations of groups. 1. In this section, some known theorems are given without proof. Most of these results, which are needed in the following, have been obtained in the theory of modular representations of groups. However, all the statements are concerned with the *ordinary* group characters.[2]

2. If $\mathfrak{G}$ is a group of order g containing k classes $K_1, \cdots, K_\mu, \cdots, K_k$ of conjugate elements, then there exist exactly k distinct irreducible characters $\zeta_1(G), \cdots, \zeta_\mu(G), \cdots, \zeta_k(G)$, where G denotes a variable element of $\mathfrak{G}$. If we restrict G to a subgroup $\mathfrak{M}$ of order m of $\mathfrak{G}$, then each $\zeta_\mu(G)$ may be considered as a (reducible or irreducible) character of $\mathfrak{M}$. From the orthogonality relations for the characters of $\mathfrak{M}$, it follows that

$$\sum{}' \zeta_\mu(G) \equiv 0 \pmod{m}, \tag{2.1}$$

where the sum extends over all elements G of $\mathfrak{M}$. More generally, the same congruence holds, if ζ is a linear combination of the ζ_μ's with coefficients which are algebraic integers.

3. Let p be a prime number and let $\mathfrak{p}$ be a prime ideal divisor of p in the algebraic number field generated by all $\zeta_\mu(G)$. Denote by $h(G)$ the number of elements in the class K_μ containing G. If ζ_μ has degree z_μ, the number $h(G)\zeta_\mu(G)/z_\mu$ is an algebraic integer. Two characters ζ_μ and ζ_ν belong to the same *p-block*, if

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[1] We use the notation of L. E. Dickson, *Linear groups*, Leipzig, 1901.

[2] The fundamental properties of group characters are given in a large number of books. Here we mention only: W. Burnside, *The theory of groups of finite order*, 2d ed., Cambridge, 1911.