does not contain any point of E_r other than p. $C_p(\eta)$ is defined as in (5). From this it can be shown that E_r has finite $(n-1)$-dimensional measure. Let us not assume now that r is large. We then write $E=\bigcup_{k=1}^{m} E^{(k)}$ where the E^k's are closed and their diameter is less than ϵ. Then, by what has been shown before, if ϵ is small enough $E_r^{(k)}$ has finite $(n-1)$-dimensional measure. Clearly $E_r \subset \bigcup_{k=1}^{m} E_r^{(k)}$. But E_r is closed, therefore its $(n-1)$-dimensional measure exists, and it clearly can not be 0, since it separates the space. Thus E_r must have finite $(n-1)$-dimensional measure.

Added in proof. The author has recently discovered that the following two theorems have been stated by C. Pauc, Revue Scientifique vol. 77 (1939) no. 8: Let the set E be in the plane then M_2 is contained in the sum of countably many Jordan curves and M_3 is countable.

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REMARK ON TAYLOR'S FORMULA

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Taylor's formula

$$(1) \qquad f(a) = \sum_{k=0}^{n-1} f^{(k)}(0)a^k/k! + f^{(n)}(\xi)a^n/n!, \qquad 0 < \xi < a,$$

is usually proved under the assumptions that

(I) $f(x)$ is continuous on the closed interval $[0, a]$;

(II) $f(x)$ possesses $n-1$ derivatives on the half closed interval $[0, a)$;

(III) $f^{(n-1)}(x)$ is continuous at $x=0$; and

(IV) $f(x)$ has an nth derivative on the open interval $(0, a)$.

In the case $n=1$, the assumption (III) that $f^{(0)}(x)\equiv f(x)$ be continuous at $x=0$ is essential but is contained in condition (I). In the case $n>1$, it will be shown below that the assumption (III) is entirely superfluous, so that (1) *is valid whenever* (I), (II) *and* (IV) *hold.*

The proof of (1) is usually reduced to an application of the mean value theorem to the $(n-1)$th derivative of $f(x)$ on an interval $[0, c]$, $0<c<a$. Thus, to prove the italicized statement, it is sufficient to show that if $f(x)$, defined on the interval $[0, a]$, is the derivative of a function and $f(x)$ itself possesses a derivative on the open interval $(0, a)$, then there exists a number ξ such that

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