If $p=q+1$, replace (21) by

$$\begin{aligned}
&[(2\alpha_k - \beta_q)(1 - x) + (A - B)x]F \\
&\qquad = \alpha_k(1 - x)F(\alpha_k +) + (\alpha_k - \beta_q)F(\alpha_k -) \\
&\qquad\quad - x\sum_{j=1}^{q-1} V_{j,k}F(\beta_j +); \qquad k = 1, 2, \cdots, p.
\end{aligned} \tag{30}$$

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# ON THE GROWTH OF THE SOLUTIONS OF ORDINARY DIFFERENTIAL EQUATIONS

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In a recent paper,[1] N. Levinson gave four theorems concerning the behaviour of the solutions of the differential equation of elastic vibrations

$$d^2x/dt^2 + \phi(t)x = 0 \tag{1}$$

as $t \to +\infty$. It is the purpose of this note to give generalizations of the Theorems I and III of Levinson by making use of certain inequalities concerning homogeneous equations of the first order

$$\frac{dx_i}{dt} + \sum_{k=1}^{n} a_{ik}x_k = 0, \qquad i = 1, \cdots, n. \tag{2}$$

Theorems I and III of Levinson run as follows:

Theorem I. *If* $\alpha(t)$ *denotes the integral*

$$\alpha(t) = \int_0^t |\phi(t) - c^2| \, dt, \tag{3}$$

*then*

$$x(t) = O\{\exp(\alpha(t)/2c)\}. \tag{4}$$

Theorem III. *If* $\alpha(t)$ *is* $O(t)$ *then*

$$\limsup_{t\to\infty} |x(t)\exp(\alpha(t)/2c)| > 0. \tag{5}$$

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[1] *The growth of the solutions of a differential equation*, Duke Math. J. vol. 8 (1941) pp. 1–11.