THE STRONG SUMMABILITY OF DOUBLE FOURIER SERIES

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1. Introduction. Corresponding to the well known theorem of Fejér-Lebesgue, we have for the double Fourier series the following proposition:

If $f\log^+|f|$ is Lebesgue integrable on the square $(-\pi\leqq x\leqq\pi, -\pi\leqq y\leqq\pi)$, then the Fejér mean $\sigma_{m,n}(x, y)$ of $f(x, y)$ tends to $f(x, y)$ almost everywhere as m and n independently increase indefinitely. Moreover, for every increasing function $\phi(t)$ satisfying the conditions

$$\phi(0) = 0, \qquad \liminf_{t\to\infty} \frac{\phi(t)}{t\log t} = 0,$$

there is a function $f(x, y)$ such that $\phi(|f|)$ is integrable and that $\sigma_{m,n}(x, y)$ does not converge almost everywhere.[1]

The latter half of this theorem shows that the analogue, in double Fourier series, of the Fejér-Lebesgue theorem is not a trivial extension of that of a function of a single variable.

The purpose of the present note is to discuss the strong summability[2] of double Fourier series. A double series $\sum a_{mn}$ is said to be strongly summable with the positive index k if there exists a constant s such that the expression

$$\frac{1}{(m+1)(n+1)}\sum_{\mu=0}^{m}\sum_{\nu=0}^{n}|s_{\mu,\nu}-s|^k \tag{1.1}$$

has the double limit zero as m and n increase without limit, where

$$s_{m,n} = \sum_{\mu=0}^{m}\sum_{\nu=0}^{n} a_{\mu\nu}.$$

It is easily seen from Hölder's equality that the summability says more for larger k.

Suppose now that $f(x, y)$ is integrable in the Lebesgue sense over

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[1] B. Jessen, J. Marcinkiewicz and A. Zygmund [5]. The first example of a function $f(x, y)\in L$ with Fejér mean divergent everywhere was given by A. Zygmund; see S. Saks [8]. Numbers in brackets refer to the Bibliography at the end of the paper.

[2] A notion first introduced in Fourier series by G. H. Hardy and J. E. Littlewood [1]. For subsequent researches, see Hardy and Littlewood [2, 3], J. Marcinkiewicz [6] and A. Zygmund [12].