ON SURFACES OF CLASS K_1

J. W. T. YOUNGS

The purpose of this note is to make a comment bearing upon the remarkable results of Radó on the semi-continuity of double integrals in parametric form.[1]

The essence of the situation, without at first attempting precision, is this. A continuous surface is of class K_1 if and only if it has a representation for which the classical double integral area formula has meaning. (It is understood that the integration is in the sense of Lebesgue.) This class of continuous surfaces is variously employed in Radó's paper. The primary object of this note is to show that *every surface is of class* K_1.

In putting things more precisely it is both convenient and economical to treat the matter in its true light; namely, as a corollary to Radó's paper. Thus the notation and terminology here follow that of Radó, and numbers in parentheses refer to the appropriate paragraphs in his paper.

To preserve a certain measure of continuity a few of the salient concepts are here reviewed.

A *continuous surface* S (1.21) is, by definition, an equivalence class of *triples of continuous functions* (1.6). If the definition of the equivalence relation is strengthened by the addition of an *orientation* requirement, then any one of the resulting equivalence classes is known as an *oriented continuous surface* ${}_oS$ (1.23). In each case any triple in the equivalence class is known as a *representation* of the surface.

The notation (T, B) is used generically to denote a *continuous triple of functions.*

$$T: \quad x^i(u^1, u^2), \qquad (u^1, u^2) \in B,\ i = 1, 2, 3,$$

where B is the closure of some Jordan region in the plane. The abbreviated notation

$$T: \quad x(u), \qquad u \in B,$$

is also employed (1.6).

If a continuous triple (T, B) is such that the six partial derivatives exist almost everywhere in B^0, then the Jacobians are denoted by

$$X^1(u^1, u^2) = \frac{\partial(x^2, x^3)}{\partial(u^1, u^2)}, \qquad X^2(u^1, u^2) = \frac{\partial(x^3, x^1)}{\partial(u^1, u^2)},$$

Presented to the Society, November 27, 1943; received by the editors June 2, 1945.

[1] Tibor Radó, *On the semi-continuity of double integrals in parametric form*, Trans. Amer. Math. Soc. vol. 51 (1942) pp. 336–361.