SOME INVARIANTS OF CERTAIN PAIRS OF HYPERSURFACES

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Introduction. It is known [8, 9][1] that if two surfaces in ordinary space have a common tangent plane at an ordinary point, then the ratio of their total curvatures at this point is a projective invariant, and the theorem holds true similarly for hyperspaces.[2] In connection with this theorem and the investigation of Bouton [2], Buzano [3] and Bompiani [1] have shown the existence of a projective invariant, together with metric and projective characterizations, determined by the neighborhood of the second order of two surfaces S, S^* at two ordinary points O, O^* in ordinary space under the conditions that the tangent planes of the surfaces S, S^* at the points O, O^* be distinct and have OO^* for the common line. Furthermore, the other case in which the tangent planes of the surfaces S, S^* at the points O, O^* are coincident[3] has been considered in recent papers of the author [6, 7].

It is the purpose of the present paper to generalize the results of the two cases mentioned above.

Let V_{n-1}, V_{n-1}^* be two hypersurfaces in a space S_n of n dimensions, and t_{n-1}, t_{n-1}^* the tangent hyperplanes of the hypersurfaces V_{n-1}, V_{n-1}^* at two ordinary points O, O^*. For the subsequent discussion it is convenient to assume in Chapter I that the tangent hyperplanes t_{n-1}, t_{n-1}^* are coincident. We can (§1), as in ordinary space, determine a projective invariant by the neighborhood of the second order of the hypersurfaces V_{n-1}, V_{n-1}^* at the points O, O^*; and the projective and metric characterizations of this invariant are given in the next two sections.

Chapter II treats of the case in which the tangent hyperplanes t_{n-1}, t_{n-1}^* are distinct, and the common tangent flat space t_{n-2} of t_{n-1}, t_{n-1}^* contains the line OO^*. We first (§4) show by analysis the existence of two projective invariants determined by the neighbor-

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[1] Numbers in brackets refer to the bibliography at the end of the paper.

[2] The simple projective characterizations of this invariant were given by C. Segre [10] for two plane curves and by P. Buzano [4] for two surfaces in space S_n ($n>2$). On the other hand, A. Terracini [11] also interpreted projectively this invariant by virtue of the conception of density of dualistic correspondences.

[3] It should be noted that for two plane curves having a common tangent at two ordinary points no projective invariant can be determined by the neighborhood of the second order of the two curves at these points. See my paper [5].