QUADRICS ASSOCIATED WITH A CURVE ON A SURFACE

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1. Introduction. Many of the important contributions to projective differential geometry of non-ruled surfaces are concerned with systems of quadrics associated with a point and a curve on the surface. Many of these quadrics belong to a certain family, a characterization of which is the main purpose of this paper.

Let the homogeneous projective coordinates (x^1, x^2, x^3, x^4) of a general point x on a non-ruled surface S be given as functions of the asymptotic parameters u, v, and let these functions be so normalized that they satisfy the Fubini canonical system of differential equations,

$$x_{uu} = \theta_u x_u + \beta x_v + px,$$
$$x_{vv} = \gamma x_u + \theta_v x_v + qx, \qquad \theta = \log(\beta\gamma),$$

wherein the coefficients satisfy certain integrability conditions [7].[1] The abbreviations

$$\phi = \partial \log(\beta\gamma^2)/\partial u, \qquad \psi = \partial \log(\beta^2\gamma)/\partial v$$

will be found useful.

Let C_λ, a curve on S through x, be considered as imbedded in a one-parameter family of curves defined by the differential equation

$$dv - \lambda du = 0.$$

Since the homogeneous coordinates of any point X may be written in the form

$$X = x_1 x + x_2 x_u + x_3 x_v + x_4 x_{uv},$$

the coordinates of X referred to the tetrahedron x, x_u, x_v, x_{uv} may be taken as (x_1, x_2, x_3, x_4).

It is remarkable that many of the equations of quadrics associated with S and C_λ at x are of the form

$$x_2 x_3 + T x_4 = 0 \tag{1}$$

wherein

$$T = -x_1 + k_2 x_2 + k_3 x_3 + k_4 x_4, \tag{2}$$

and

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[1] Numbers in brackets refer to the references cited at the end of the paper.