A CLASS OF INTEGRAL EQUATIONS WHICH GENERALIZE ABEL'S EQUATION

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Introduction. A general class of integral equations is solved in §1. Specific examples are given in §3 which are shown in §5 to specialize to an equation from which Abel's may readily be derived. In this sense the results of this paper generalize Abel's integral equation solution. Another interesting special case is given in §6.

1. **A general theorem.** Our development depends on this theorem:

THEOREM. (i) *Let the function $g(x)$ possess a continuous derivative for $x>0$, and let $\int^\infty g(t)tdt$ converge. Let $p(x)$, $q(x)$ be functions for which*

$$\int_0^\infty\int_0^\infty |g((x^2+s^2+t^2)^{1/2})||p(s)||q(t)|dsdt \text{ exists,} \tag{1}$$

and for which the condition

$$\int_0^{\pi/2} p(r\cos\theta)q(r\sin\theta)d\theta = 1 \tag{2}$$

holds. Then

$$\phi(x) = f(x) \equiv -\frac{1}{x}\frac{d}{dx}\int_0^\infty g((x^2+t^2)^{1/2})q(t)dt \tag{3}$$

is a solution of the integral equation

$$g(x) = \int_0^\infty \phi((x^2+t^2)^{1/2})p(t)dt. \tag{4}$$

(ii) *If $\phi(x)$ is a solution of* (4) *having a continuous derivative for $x>0$, such that $\int^\infty\phi(t)tdt$ converges and such that*

$$\int_0^\infty\int_0^\infty |\phi((x^2+s^2+t^2)^{1/2})||p(s)||q(t)|dsdt \text{ exists,} \tag{5}$$

then

$$\phi(x) = f(x).$$

PROOF. From (3),

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