ON UNIFORM CONVERGENCE OF TRIGONOMETRIC SERIES

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1. **Introduction.** The following theorems have been proved previously.[1]

THEOREM I. *If the function* $\phi(t)$ *is throughout continuous, periodic of period* 2π, $\phi(t)=\phi(-t)=\phi(2\pi+t)$,

$$\phi(t) \sim \frac{a_0}{2} + \sum_1^\infty a_n \cos nt, \tag{1.1}$$

and if

$$na_n > -K, \tag{1.2}$$

for some constant K, *and all* n, *then the series* (1.1) *is uniformly convergent* (*on the real axis*).

THEOREM II. *If* $f(t)$ *is everywhere continuous, periodic of period* 2π' $f(t)=-f(-t)$,

$$f(t) \sim \sum_1^\infty b_n \sin nt, \tag{1.3}$$

and if

$$nb_n > -K, \qquad n = 1, 2, 3, \cdots, \tag{1.4}$$

then the series (1.3) *is uniformly convergent.*

THEOREM III (CHAUNDY AND JOLLIFFE). *The Fourier series* (1.3) *is uniformly convergent, if*

$$b_n \geqq b_{n+1} > 0, \quad \text{and if} \quad nb_n \to 0. \tag{1.5}$$

Note that here no explicit assumption is made on $f(t)$.

THEOREM IV. *If* $\phi(t)$ *is continuous at* $t=0$, *and if*

$$\lim_{\lambda \downarrow 1} \limsup_{n\to\infty} \sum_n^{\lambda n} (|a_\nu| - a_\nu) = 0, \tag{1.6}$$

then the series (1.1) *is uniformly convergent at* $t=0$. (*That is,* $s_n(t_n)\to s$ *whenever* $t_n \to 0$.)

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[1] Cf. [2] and the references given there; numbers in brackets refer to the literature cited at the end of this paper.