ABSTRACTS OF PAPERS

SUBMITTED FOR PRESENTATION TO THE SOCIETY

The following papers have been submitted to the Secretary and the Associate Secretaries of the Society for presentation at meetings of the Society. They are numbered serially throughout this volume. Cross references to them in the reports of the meetings will give the number of this volume, the number of this issue, and the serial number of the abstract.

Algebra and Theory of Numbers

260. R. A. Beaumont: *Groups with isomorphic proper subgroups.*

A completely reducible group is a group which is the direct product of groups of rank 1. Necessary and sufficient conditions that a completely reducible group have an isomorphic proper subgroup are found. If G is a completely reducible abelian group without elements of infinite order, then G has an isomorphic proper subgroup if, and only if, G is of infinite rank. If G is a completely reducible abelian group without elements of finite order, then G has an isomorphic proper subgroup if, and only if, either G has a direct factor which is essentially a proper subgroup of the additive group of all rational numbers or G is of infinite rank. (Received September 29, 1944.)

261. R. H. Bruck: *Quasigroup theory.* II. *The lower central series.* Preliminary report.

This abstract is a continuation of abstract 50-5-107. Let G be a loop with unit 1, $\mathfrak{G}$ be the group consisting of all products of a finite number of permutations R_x, L_y, R_z^{-1} and L_u^{-1}, where $xy = xR_y = yL_x$. R. Baer has posed the problem of determining an "inner mapping group" $\mathfrak{I} \subset \mathfrak{G}$ with the property that a subloop $H \subset G$ is normal if and only if invariant under $\mathfrak{I}$. It is shown here that $\mathfrak{I}$ is generated by the set of all permutations $R_{x,y} = R_x R_y R_{xy}^{-1}$, $M_{x,y} = R_y L_x R_{xy}^{-1}$; moreover $U \subset \mathfrak{G}$ is in $\mathfrak{I}$ if and only if $1U = 1$. If H is any subloop of G and $\mathfrak{M}$ any set of mappings of G which map H into itself, the H-subloop $H(\mathfrak{M})$ generated by all elements xAL_x^{-1} with $x \subset H$, $A \subset \mathfrak{M}$ is mapped into itself by $\mathfrak{M}$. In particular if H is a normal subloop of G then $H(\mathfrak{I})$ is normal both in G and in H, and $H/H(\mathfrak{I})$ is an abelian group. The lower central series of G may be defined by $H_0 = G$, $H_1 = G' = H_0(\mathfrak{I})$, $H_2 = H_1(\mathfrak{I})$, $\cdots$, and the notions of solubility and class are immediate. $\mathfrak{I}$ is a group of automorphisms of G only in special cases, as for example in the (non-trivial) case of a commutative Moufang loop. (Received August 21, 1944).

262. R. H. Bruck: *Quasigroup theory.* III. *Finite p-loops.* Preliminary report.

In this study a finite loop G is called a p-loop if and only if the associated group $\mathfrak{G}$ is a p-group. Also if H is a normal subloop of G then by definition $(H, G) = H(\mathfrak{I})$. (For explanation of these notations see abstract 50-11-261.) Finally a loop is said to be abelian if and only if it is a commutative group. It is then shown that many of the elementary theorems for p-groups, and even some of the proofs, remain valid when