Statistics and Probability

248. C. W. Churchman and Benjamin Epstein: *Statistics of sensitivity data.* II. Preliminary report.

In this paper a study is made of the distribution of the first two moments of sensitivity data as functions of the sample size. The chief results are briefly these: (a) The distributions of $\bar{x}$ and σ^2 (for definition of these functions, see *On the statistics of sensitivity data* by the authors in the Annals of Mathematical Statistics vol. 15 (1944)) approach normality rapidly as functions of the sample size; (b) $\bar{x}$ and $\sigma_{\bar{x}}^2$ are "almost" independent even for small sample sizes, thus justifying the use of Student's ratio in tests of significance for differences between two sample means. (Received July 1, 1944.)

249. E. J. Gumbel: *Ranges and midranges.*

The mth range w_m and the mth midrange $\mathfrak{r}_m$ are defined as the difference and as the sum of the mth extreme value taken in descending magnitude ("from above") and the mth extreme value taken in ascending magnitude ("from below"). The semi-invariant generating functions $L_m(t)$ and ${}_mL(t)$ of the mth extreme values from above and from below are simple generalizations of the semi-invariant generating functions of the largest and of the smallest value which have been given by R. A. Fisher and L. H. C. Tippett. If the sample size is large enough the two mth extreme values may be considered as independent variates. Then the semi-invariant generating functions $L_w(t, m)$ and $L_{\mathfrak{r}}(t, m)$ of the mth range and of the mth midrange are $L_w(t, m) = L_m(t) + {}_mL(-t)$; $L_{\mathfrak{r}}(t, m) = L_m(t) + {}_mL(t)$. If the initial distribution is symmetrical the semi-invariant generating function of the mth range is twice the semi-invariant generating function of the mth extreme value from above. The distribution of the mth range is skew, whereas the distribution of the mth midrange is of the generalized, symmetrical, logistic type. The even semi-invariants of the mth midrange are equal to the even semi-invariants of the mth range. For increasing indices m the distributions of the mth extremes, of the mth ranges, and of the mth midranges converge toward normality. (Received July 14, 1944.)

250. P. L. Hsu: *The approximate distribution of the mean and of the variance of independent variates.*

Let X_k be mutually independent random variables with the same cumulative distribution function; let $E(X_k) = 0$, $E(X_k^2) = 1$ and $E(X_k^4) = \delta$. Finally put $S = n^{-1} \sum_{k=1}^n X_k$ and $\eta = n^{-1} \sum_{k=1}^n (X_k - S)^2$. The author first gives a new derivation of H. Cramér's well known asymptotic expansions for Pr $(n^{1/2}S \leqq x)$. The proof is much more elementary and avoids in particular the use of M. Riesz' singular integrals. Instead a considerably simpler Cesàro-type kernel is used, which has first been introduced by A. C. Berry (Trans. Amer. Math. Soc. vol. 49 (1941) pp. 122–136). The same method is then used to derive similar asymptotic expansions for Pr $(n^{1/2}(\eta - 1) \leqq (\delta - 1)^{1/2} x)$. The method can be extended to the case of unequal components and also for the study of other functions encountered in mathematical statistics. (Received July 3, 1944.)

251. F. E. Satterthwaite: *Error control in matrix calculation.*

The arithmetic evaluation of matrix expressions is often rather complicated. One of the causes of this is the fact that relatively minor errors (such as rounding errors)