SIMPLIFIED TECHNIQUE FOR CONSTRUCTING ORTHONORMAL FUNCTIONS

M. O. PEACH

1. Introduction. An orthonormalization process starts with a set of linearly independent functions

$$f_1, f_2, \cdots,$$

and the set of complex conjugate functions

$$\bar{f}_1, \bar{f}_2, \cdots,$$

all defined over a given region R. From these are constructed a set of functions

$$g_1, g_2, \cdots,$$

and the set of complex conjugate functions

$$\bar{g}_1, \bar{g}_2, \cdots,$$

defined over R and such that

$$\int_R g_m \bar{g}_n dR = \begin{cases} 0, & \text{if } m = n, \\ 1, & \text{if } m \neq n. \end{cases}$$

The standard method[1] of constructing orthonormal functions, while completely satisfying logically, has certain practical disadvantages. For example, if the integrations must be done numerically (as would be necessary if either the f_i or the boundary of R were complicated functions, or if the f_i were tabular functions) then the mere tabulation of the intermediate functions which appear becomes burdensome. One would prefer to perform the necessary integrations on the original functions f_i and then proceed by a purely algebraic or numerical process to obtain the g_i. This can be done. If we let N_i be the numerator and D_i the denominator of the orthonormal function g_i, and if we put $F_{ij} = \int_R f_i \bar{f}_j dR$, then the standard orthonormalization process can be shown, by simple algebra, to result in the following:

$$N_1 = f_1, \qquad D_1^2 = F_{11},$$

$$N_2 = \begin{vmatrix} F_{11} & f_1 \\ F_{21} & f_2 \end{vmatrix}, \quad D_2^2 = F_{11} \cdot \begin{vmatrix} F_{11} & F_{12} \\ F_{21} & F_{22} \end{vmatrix},$$

Presented to the Society, September 13, 1943; received by the editors August 18, 1943, and, in revised form, December 8, 1943.

[1] Courant and Hilbert, *Methoden der matematischen Physik*, vol. 1, p. 41.