# ON THE GROWTH OF SOLUTIONS OF LINEAR DIFFERENTIAL EQUATIONS

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1. **Introduction.** In a recent paper by Boas, Boas and Levinson [1][1] two sets of sufficient conditions were given for the existence of $\lim_{x=\infty} y'(x)$ when $y(x)$ satisfies the differential equation

$$y'' + A(x)y = B(x). \tag{1:1}$$

We propose in this paper to use their methods and to generalize their results to the $n$th order linear differential equation

$$y^{(n)} + \sum_{i=1}^{n} A_i(x)y^{(n-i)} = B(x), \tag{1:2}$$

and to obtain sufficient conditions for

$$\lim_{x=\infty} y^{(n-1)}(x) \tag{1:3}$$

to exist. In case $n=2$, $A_1(x)=0$ and $A_2(x)=A(x)$, these conditions reduce to those in [1].

2. **Statements of the theorems.** In §4 we shall prove the following theorem.

THEOREM I. *If $A_i(x)$ $(i=1, \cdots, n)$ and $B(x)$ are continuous on $0 \leqq x < \infty$, and if the integrals*

$$\int_0^\infty x^{i-1} | A_i(x) | \, dx \qquad (i = 1, \cdots, n), \tag{2:1}$$

$$\int_0^\infty B(x)dx \tag{2:2}$$

*exist, then the limit* (1:3) *exists for any solution $y(x)$ of* (1:2).

We now write each function $A_i(x)$ as the difference of two non-negative functions, $A_i(x) = A_i'(x) - A_i''(x)$, where $A_i' = (|A_i| + A_i)/2$, $A_i'' = (|A_i| - A_i)/2$. Then in §5, $\cdots$, §8 we shall prove the following theorem.

THEOREM II. *If $A_i(x)$ $(i=1, \cdots, n)$ and $B(x)$ are continuous on $0 \leqq x < \infty$, if the integrals*

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[1] Numbers in brackets refer to the Bibliography at the end of the paper.